An Exactly Solvable Model for Algebraic Spin Liquid

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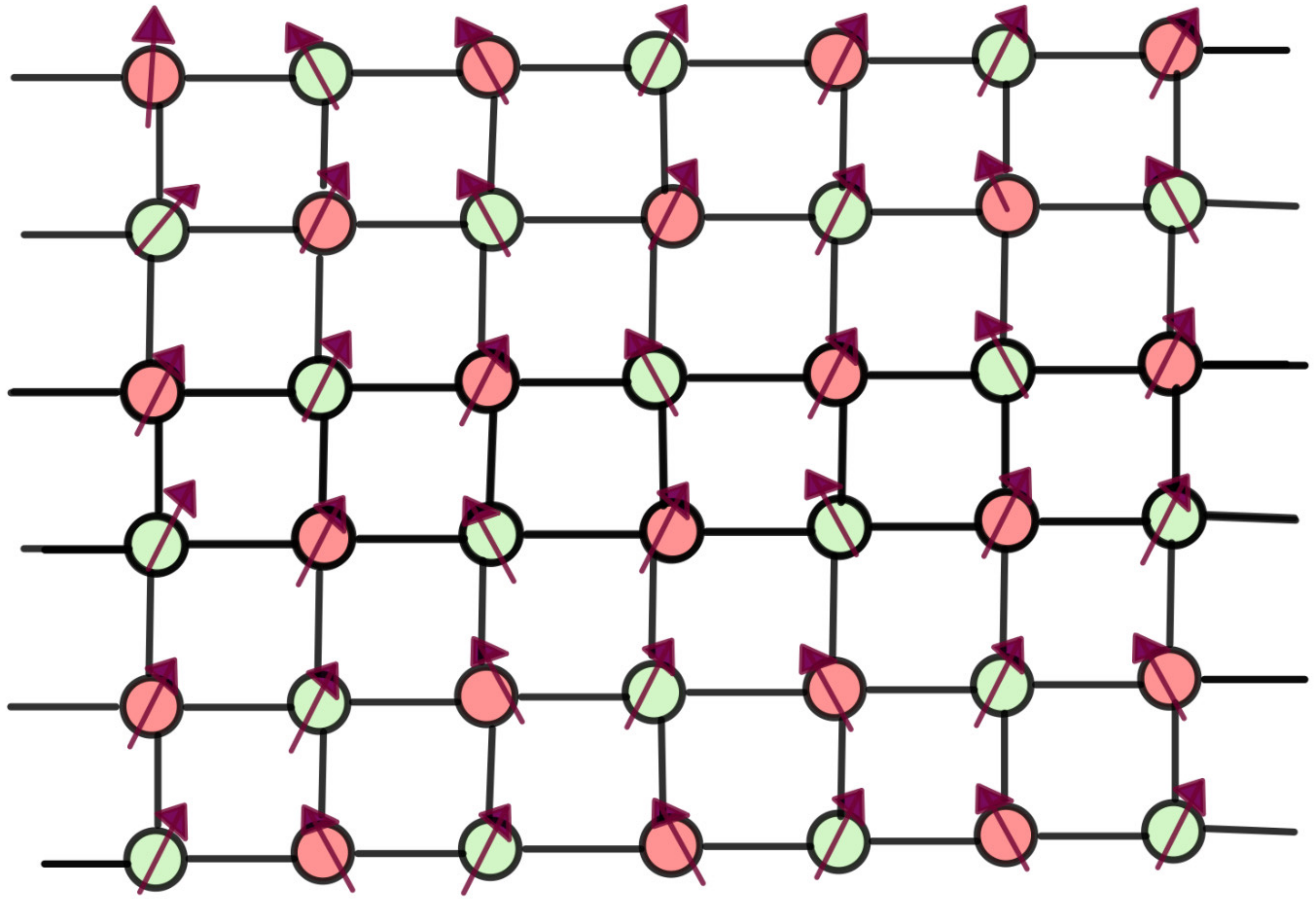


P451 (Advanced Solid-State Physics)
Term-Paper Presentation

Mott Insulator

$$H_{Hubbard} = -\sum_{ij,\alpha} t_{ij} c^\dagger_{i\alpha} c_{j\alpha} + U \sum_i n_i(n_i - 1)$$

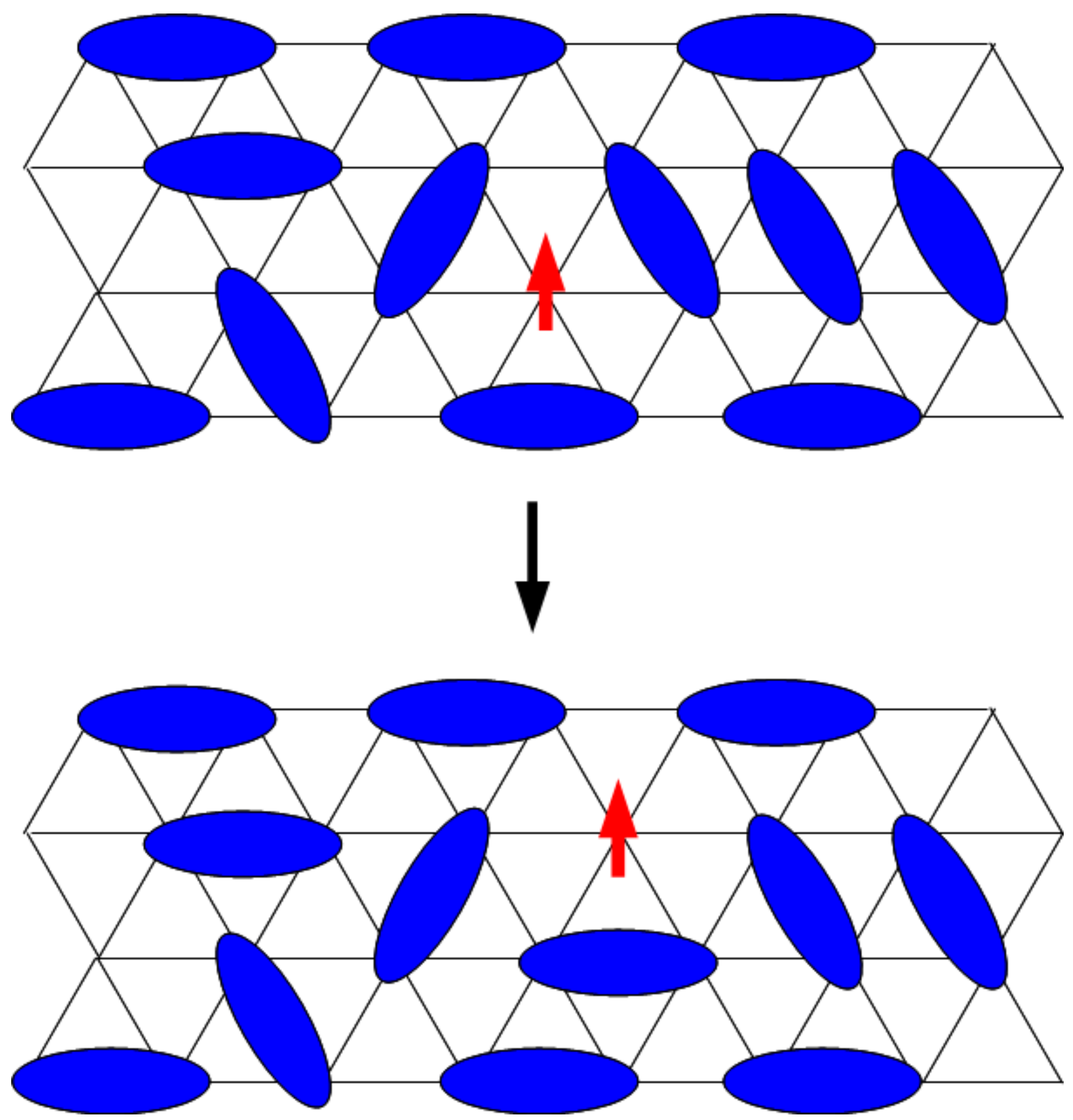
Mott Transition

$\Downarrow$

Mott Insulator

Quantum Spin Liquids

NOT symmetry broken states with conventional magnetic ordering!



Picture credit: Wikipedia.

Algebraic Spin Liquids

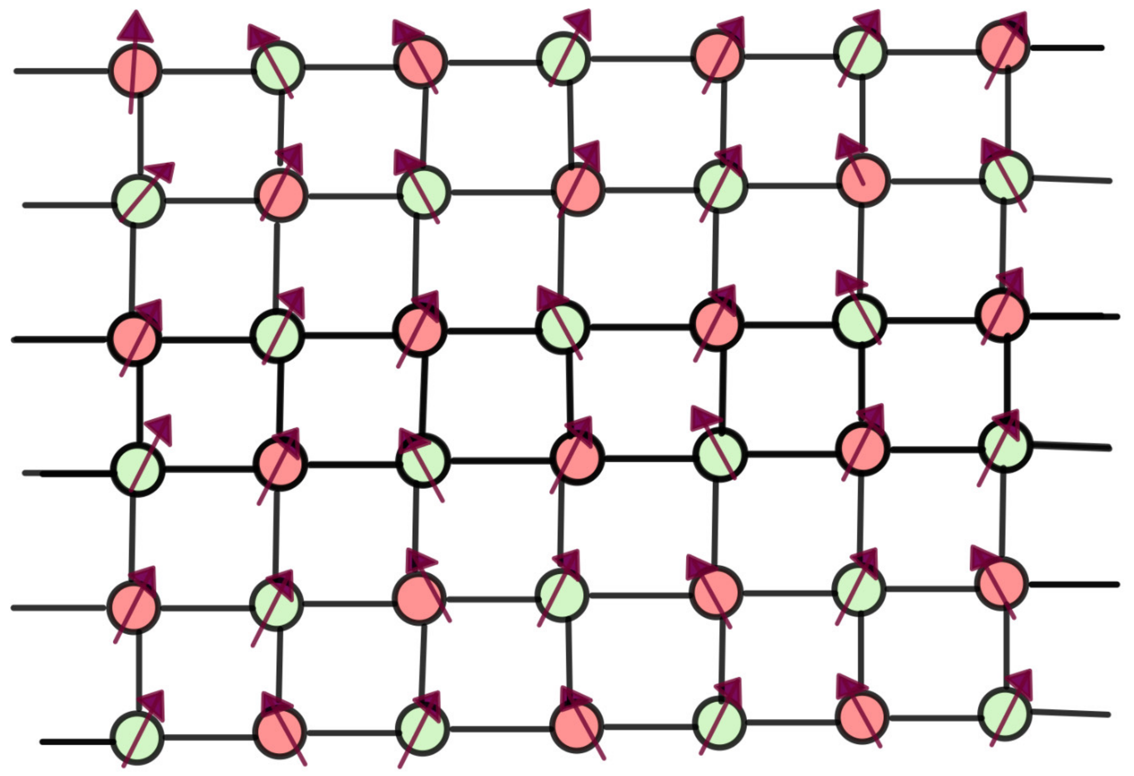
QSL with a **gapless** spinon spectrum.

Why should we care?

Spin Models

Simple, stripped down models of interactions among particles in materials.

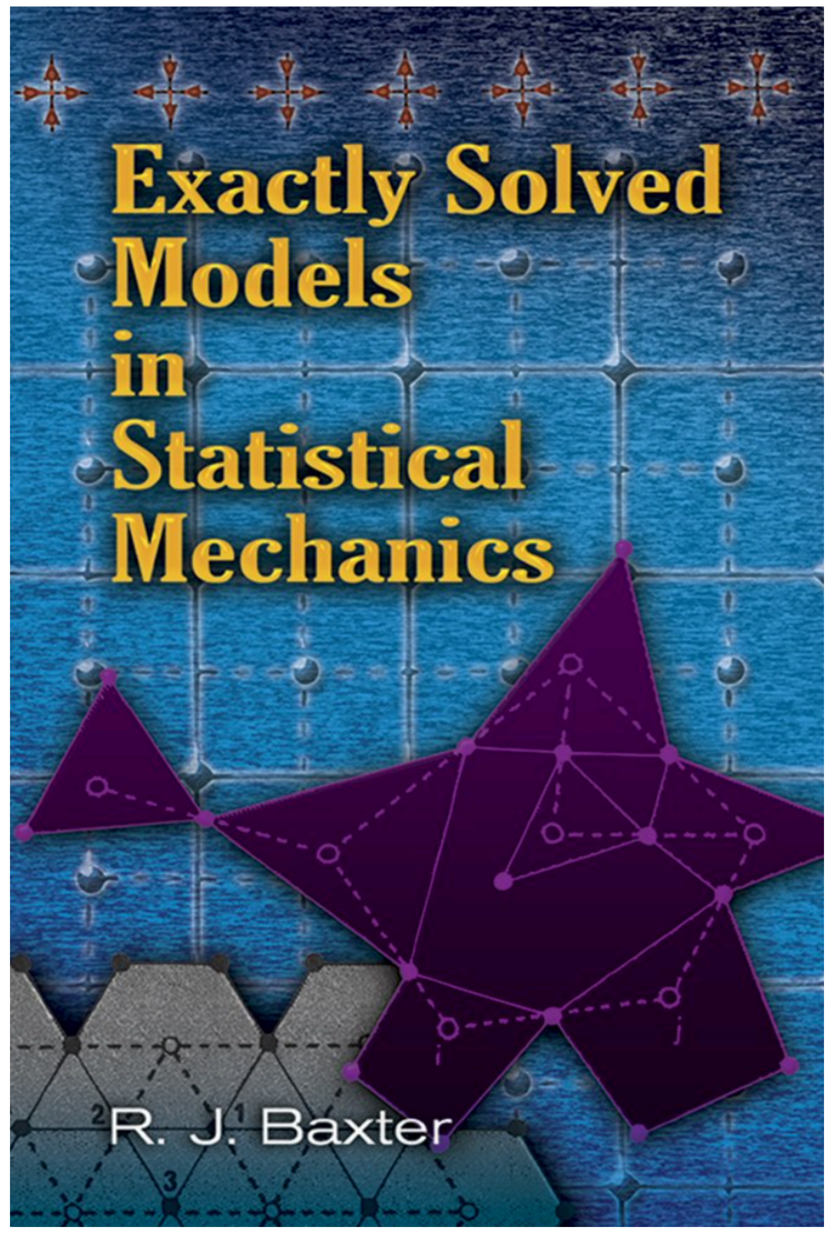
Easy to study?



Exactly Solvable Models

Exactly?

Rare!



Picture credit: Amazon.

Γ - Matrix Models

The famous Dirac-Gamma matrices satisfying the Clifford algebra.

$$\{\Gamma^a, \Gamma^b\} = 2\delta^{ab}$$

Provide a structure!

Pauli matrices also form a representation of the Gamma group.

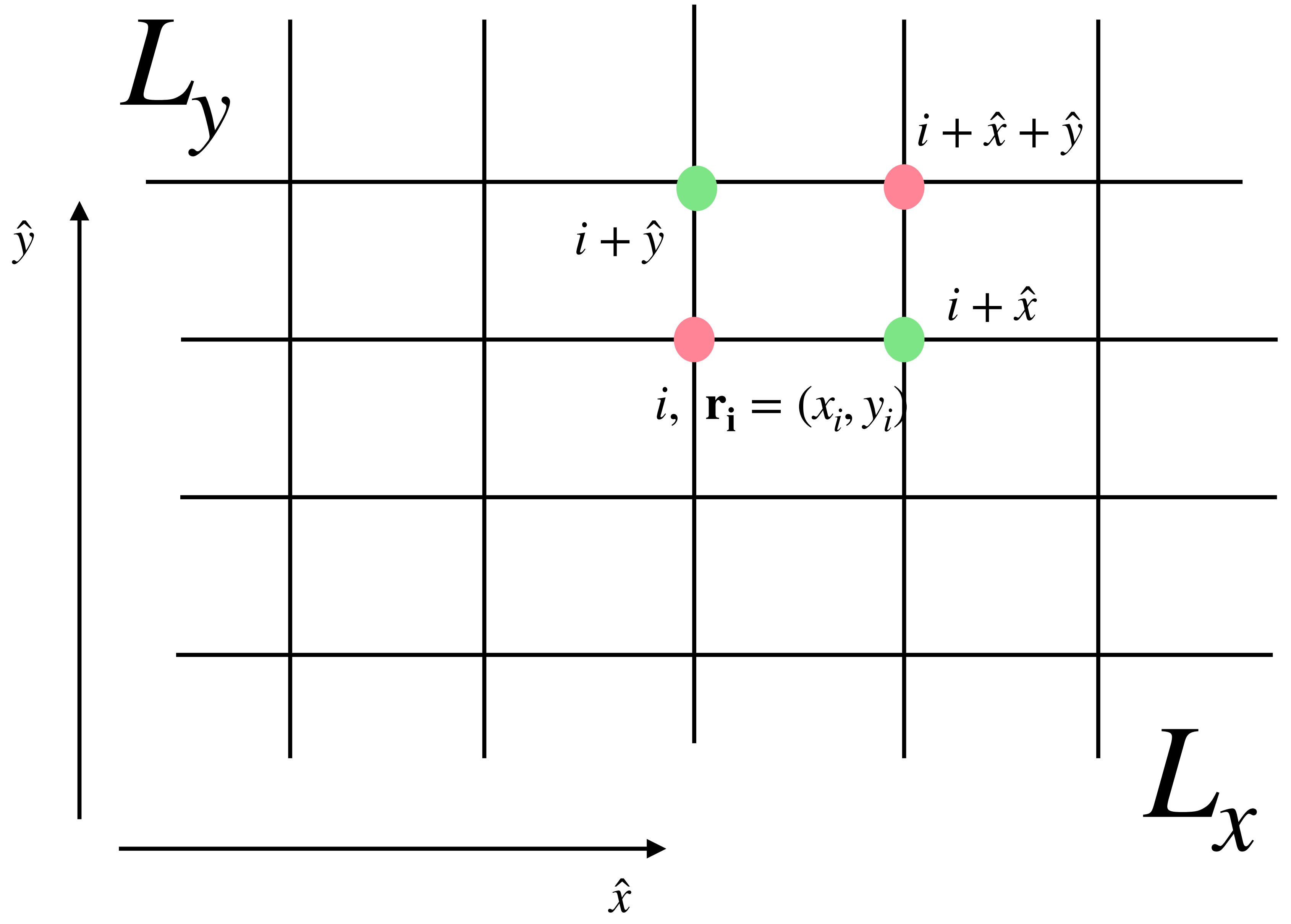
The Exactly Solvable Model

$$\mathscr{H} = \sum_i \left(J_x \Gamma_i^1 \Gamma_{i+\hat{x}}^2 + J_y \Gamma_i^3 \Gamma_{i+\hat{y}}^4 + J_x' \Gamma_i^{15} \Gamma_{i+\hat{x}}^{25} + J_y' \Gamma_i^{35} \Gamma_{i+\hat{y}}^{45} - J_5 \Gamma_i^5 \right)$$

$$\Gamma^1 = \frac{1}{\sqrt{3}}\{S^y, S^z\} \qquad \Gamma^2 = \frac{1}{\sqrt{3}}\{S^z, S^x\}$$

$$\Gamma^3 = \frac{1}{\sqrt{3}}\{S^x, S^y\} \qquad \Gamma^4 = \frac{1}{\sqrt{3}}[(S^x)^2 - (S^y)^2]$$

$$\Gamma^5 = (S^z)^2 - \frac{5}{4} \qquad \Gamma^{ab} = \frac{[\Gamma^a, \Gamma^b]}{(2\imath)}$$



$$\hat{W}_i = \Gamma_i^{13} \Gamma_{i+\hat{x}}^{23} \Gamma_{i+\hat{y}}^{14} \Gamma_{i+\hat{x}+\hat{y}}^{24}$$

$$[\hat{W}_i, \hat{W}_j] = 0$$

$$[\hat{W}_i, \mathscr{H}] = 0$$

An infinite set of conserved fluxes.

$$N = L_x L_y$$

$$N | 2$$

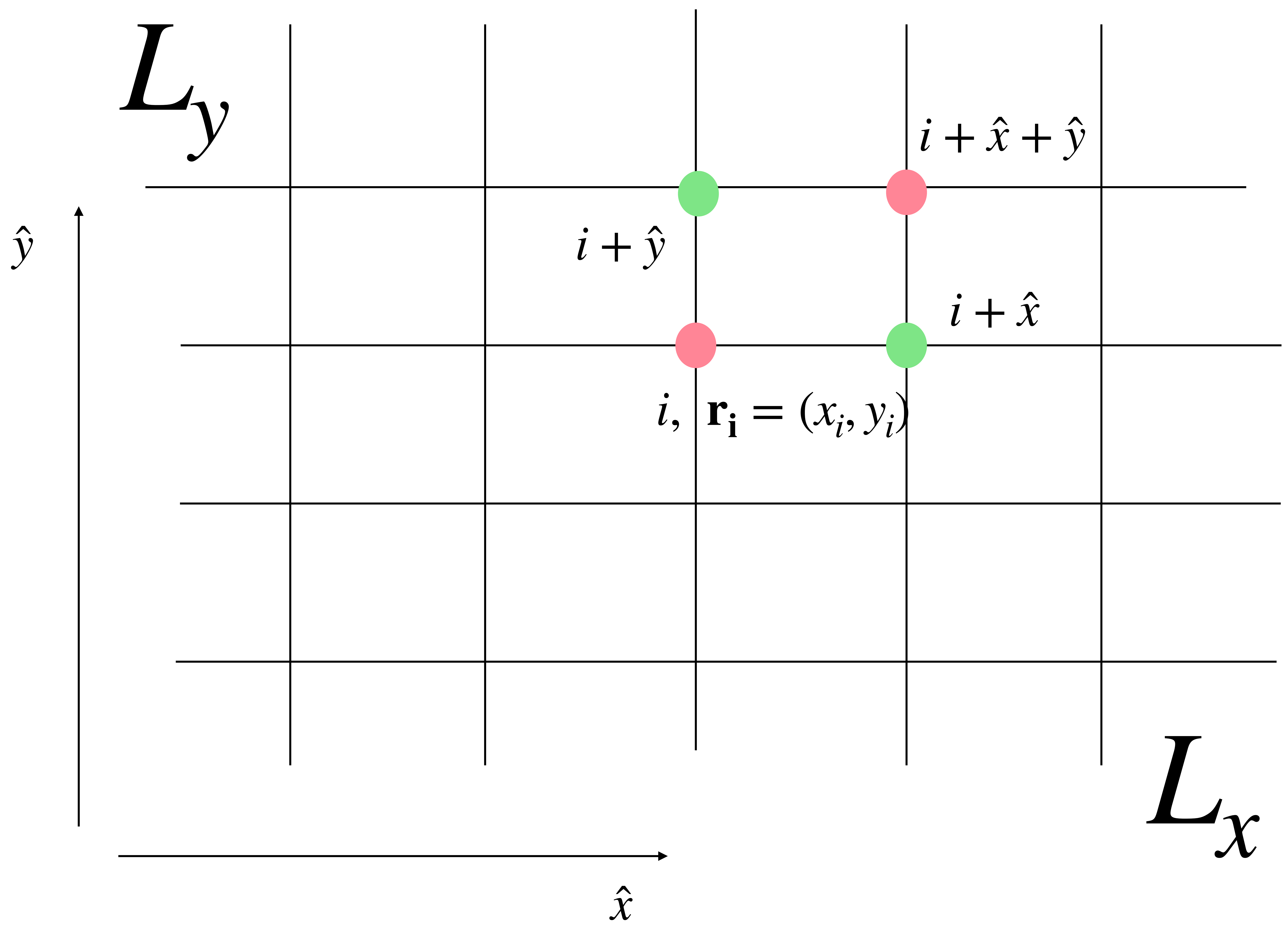
PBCs are assumed.

Symmetries

Translational Time-Reversal ✓

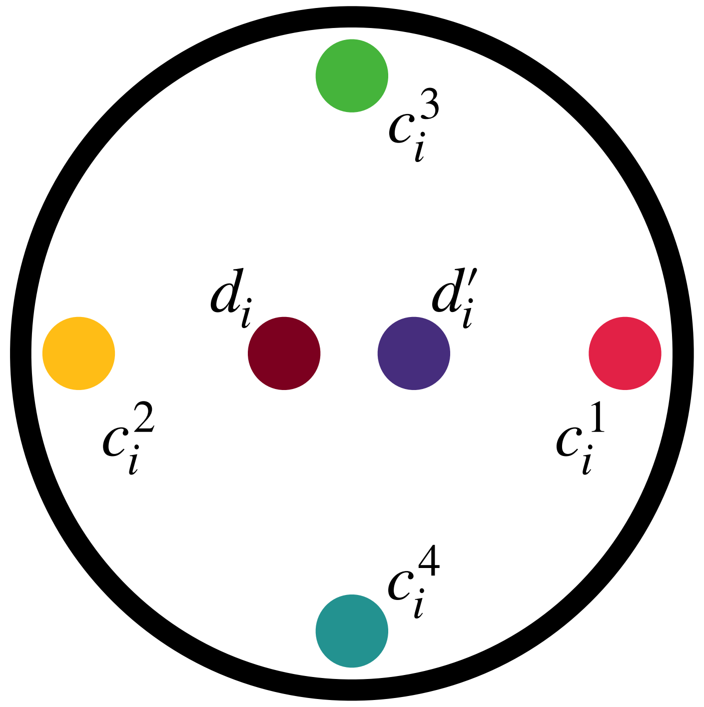
SU(2) U(1) ✗

Global Ising Symmetry ✓



L_y
$i+\hat{x}+\hat{y}$
$\hat{y}$
$i+\hat{y}$
$i+\hat{x}$
$i,\ \mathbf{r_i}=(x_i, y_i)$
L_x
$\hat{x}$

Fermionisation into Majorana Fermions



$$\Gamma_i^{\mu} = \imath c_i^{\mu} d_i \qquad \Gamma_i^{\mu 5} = \imath c_i^{\mu} d_i' \qquad \Gamma_i^{5} = \imath d_i d_i'$$

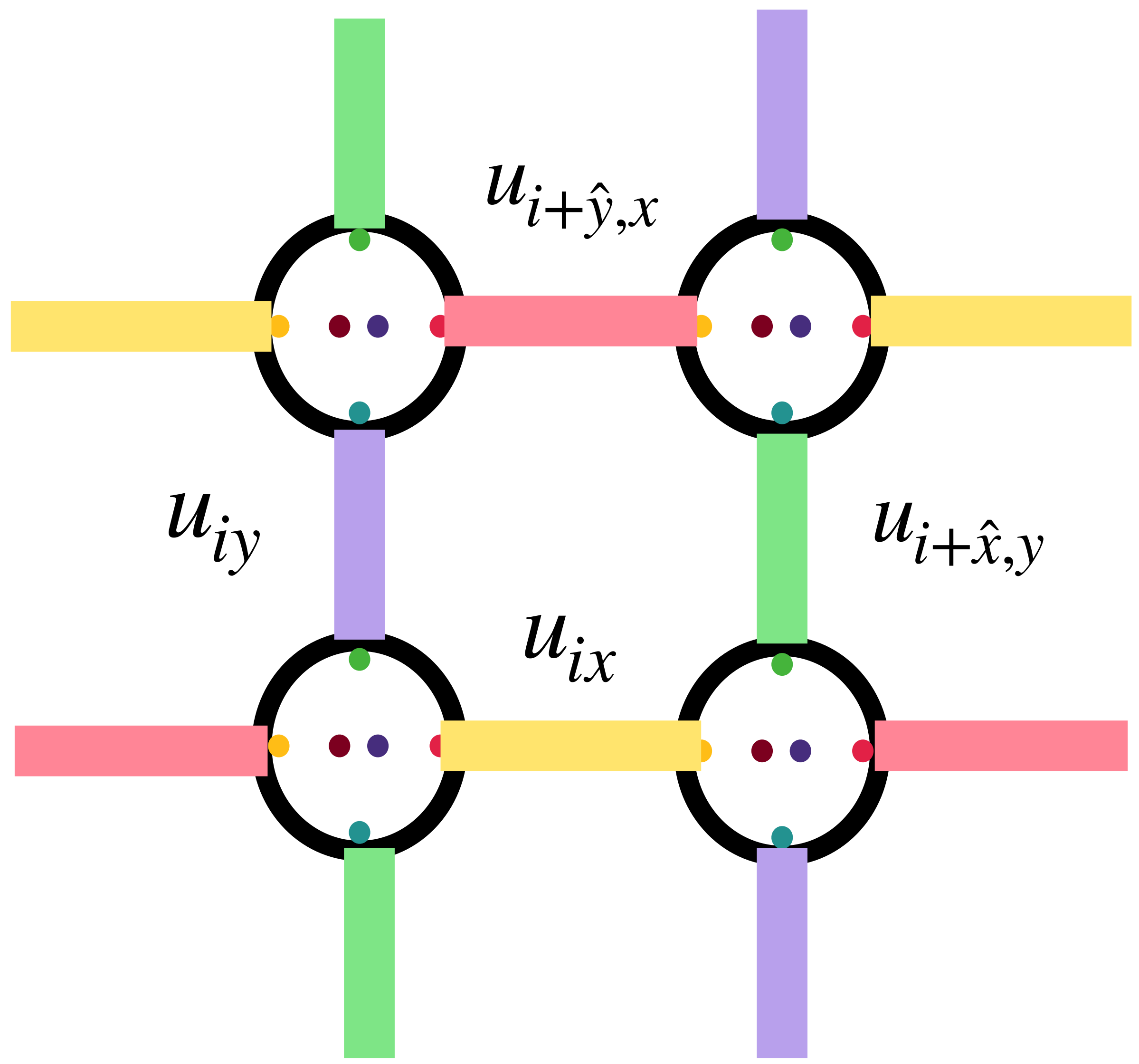
$$\mathscr{H} = \sum_i \left(J_x \hat{u}_{ix} \imath d_i d_{i+\hat{x}} + J_y \hat{u}_{iy} \imath d_i d_{i+\hat{y}} + J_x' \hat{u}_{ix} \imath d_i' d_{i+\hat{x}}' + J_y' \hat{u}_{iy} \imath d_i' d_{i+\hat{y}}' - J_5 \imath d_i d_i' \right)$$

$$\hat{u}_{ix} = -\imath c_i^1 c_{i+\hat{x}}^2 \qquad \hat{u}_{iy} = -\imath c_i^3 c_{i+\hat{y}}^4$$

$$[\hat{u}_{i\lambda}, \mathscr{H}] = 0$$

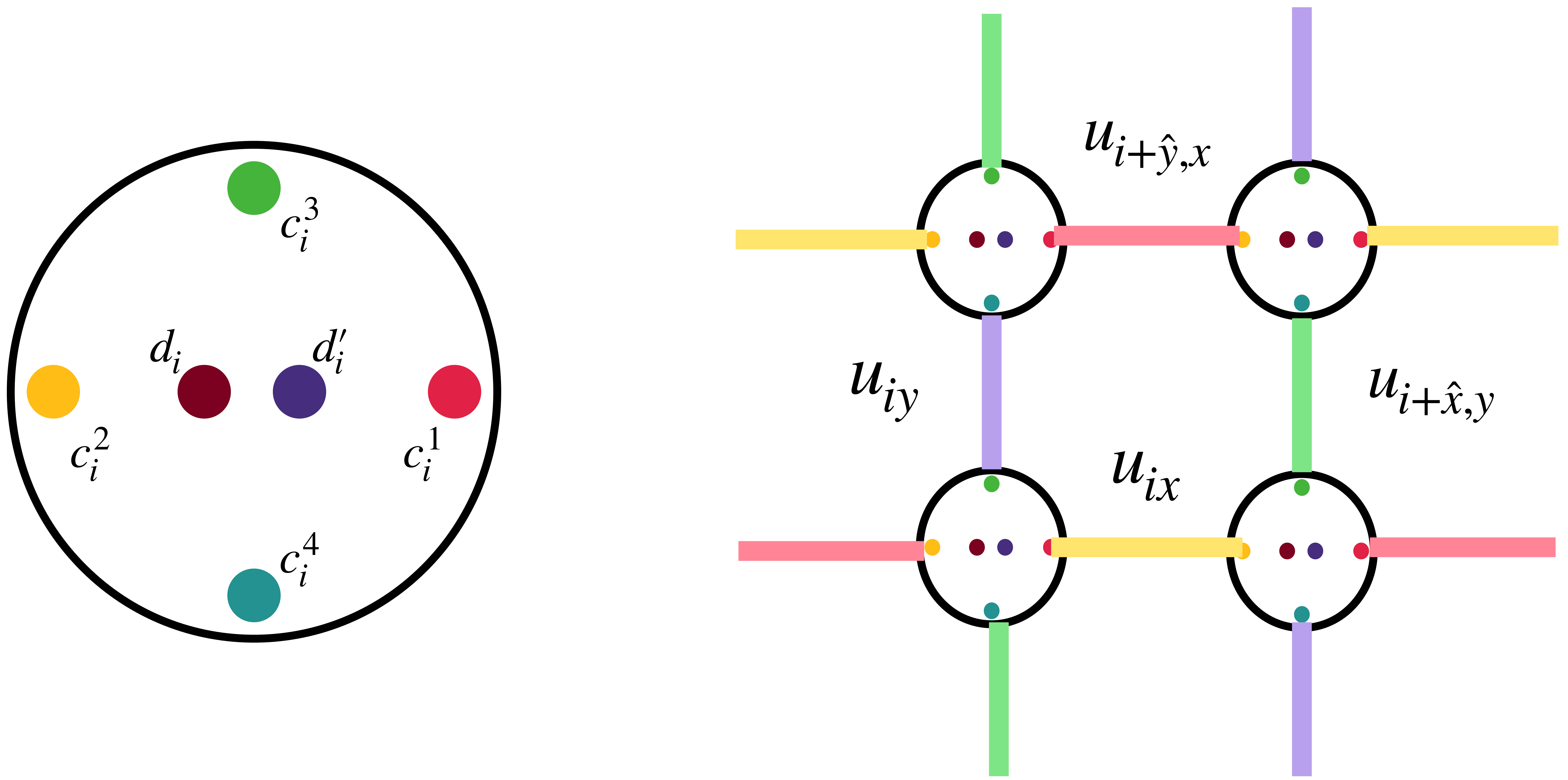
$$\left(\hat{u}_{i\lambda}\right)^2 = 1 \quad \Longrightarrow$$

$$u_{i\lambda} = \pm 1$$



$$\mathscr{H}(\{u\}) = \sum_i \left(J_x u_{ix} \imath d_i d_{i+\hat{x}} + J_y u_{iy} \imath d_i d_{i+\hat{y}} + J_x' u_{ix} \imath d_i' d_{i+\hat{x}}' + J_y' u_{iy} \imath d_i' d_{i+\hat{y}}' - J_5 \imath d_i d_i' \right)$$

$d_i' \longrightarrow \Lambda_i d_i' \qquad d_i \longrightarrow \Lambda_i d_i \qquad u_{i\lambda} \longrightarrow \Lambda_i u_{i\lambda} \Lambda_{i+\lambda}$


c_i^3
d_i
d_i'
c_i^2
c_i^1
c_i^4
$u_{i+\hat{y},x}$
u_{iy}
$u_{i+\hat{x},y}$
u_{ix}

Degeneracy of the enlarged Hilbert space

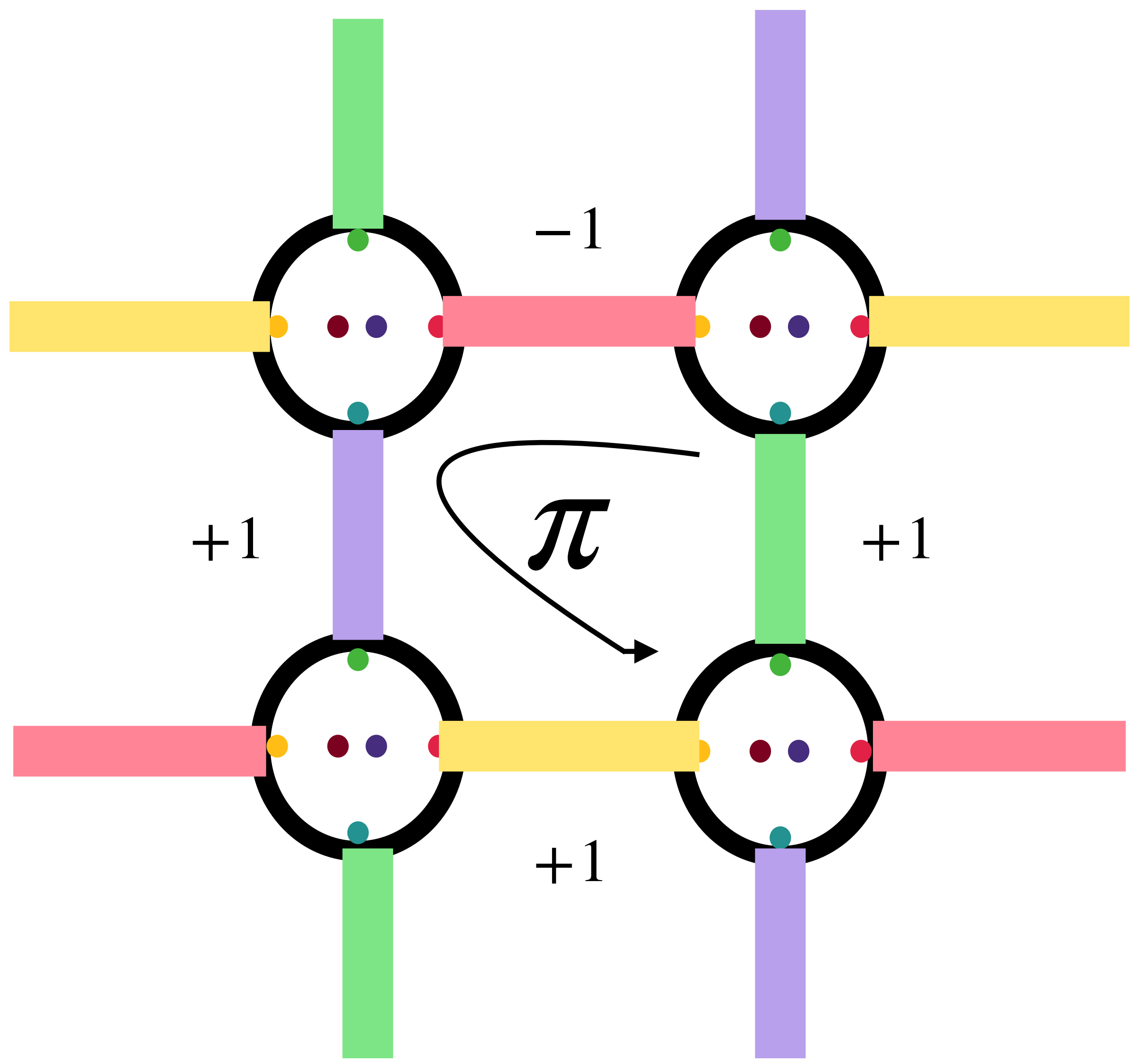
$$e^{(\imath\phi_i)} = u_{ix}u_{i+\hat{x}y}u_{iy}u_{i+\hat{y}x}$$

$$e^{\imath\phi_x} = \prod_{i(y_i=1)} u_{ix}$$

$$e^{\imath\phi_y} = \prod_{i(x_i=1)} u_{iy}$$

$$\Downarrow \quad u_{i\lambda} = \pm 1, \quad \phi_i,\ \phi_\lambda = 0,\ \pi$$

$$\sum_i \phi_i = 0\ (mod\ 2\pi)$$



Independent fluxes = N+1

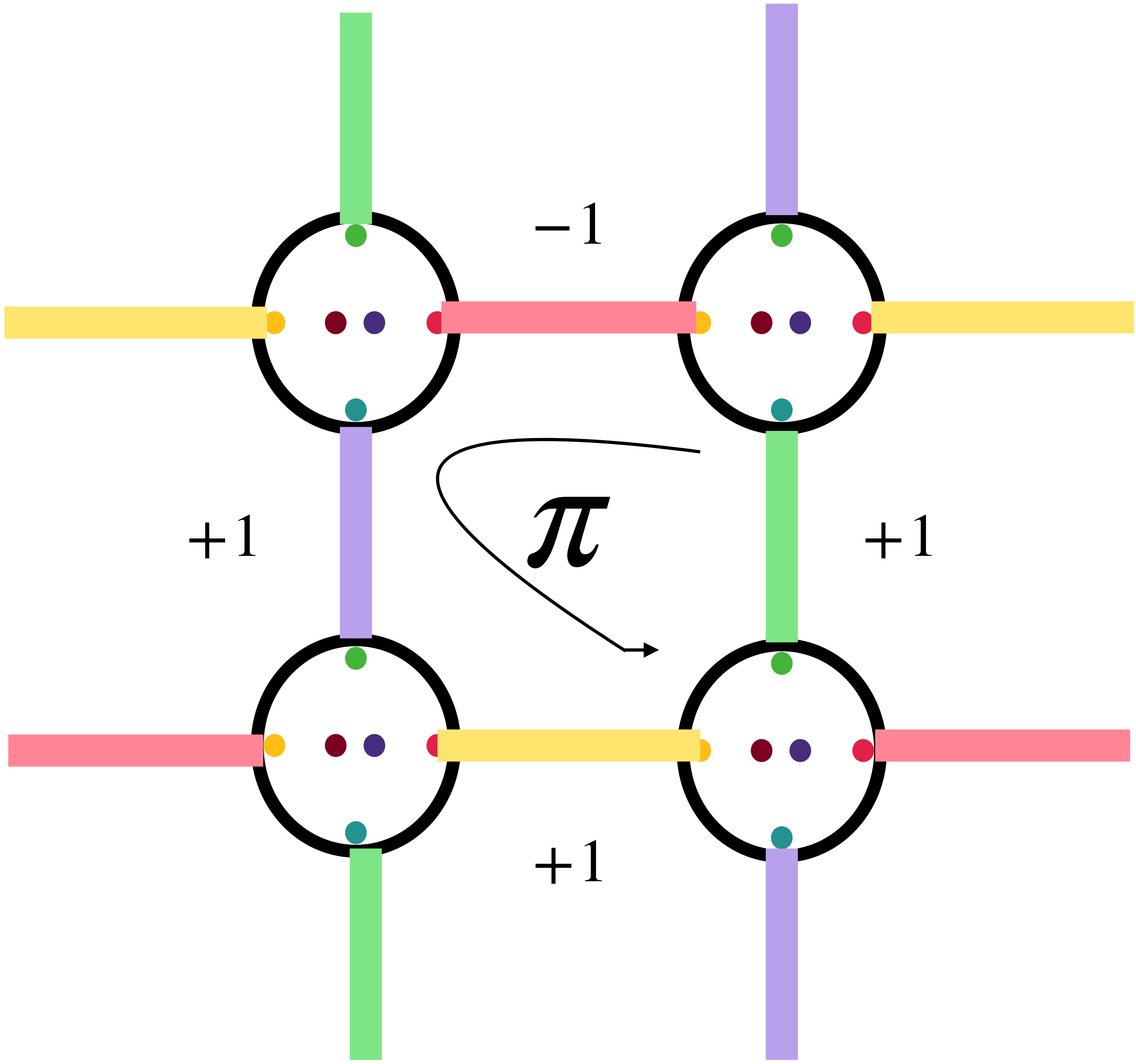
Total number of gauge fields fluxes = 2N

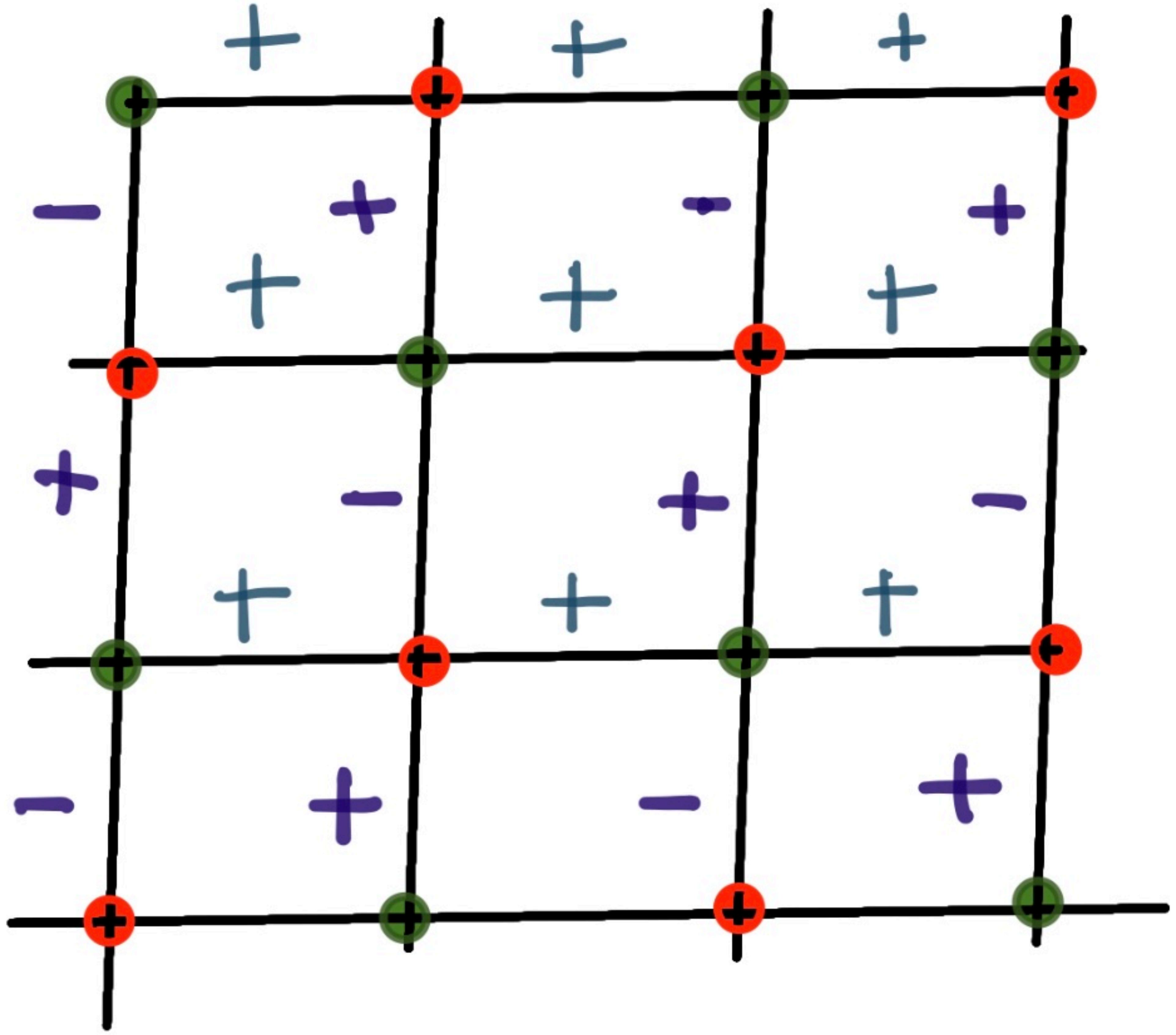
$$\frac{2^{2N}}{2^{N+1}} = 2^{N-1}$$

(Number of different gauge field choices)

$$\Downarrow$$

2^{N-1}-fold degeneracy of each state in the enlarged Hilbert space!


−1
+1
π
+1
+1



Projection to Physical States

$$\Gamma_i^1\Gamma_i^2\Gamma_i^3\Gamma_i^4\Gamma_i^5 = -1$$

$$D_i = -\imath c_i^1 c_i^2 c_i^3 c_i^4 d_i d_i'$$

$[D_i, \mathscr{H}] = 0$

$D_i^2 = 1$

+1 physical states

-1 unphysical states

$$\hat{P} = \prod_i \left[\frac{1+D_i}{2}\right]$$

$$\hat{P} = \frac{\left[1 + \sum_i D_i + \sum_{i_1<i_2} D_{i_1}D_{i_2} + \cdots + \prod_i D_i\right]}{2^N}$$

$[D, \mathscr{H}] = 0$

$D^2 = 1$

+1 physical states

-1 unphysical states

$$\hat{P} = P'\frac{(1+D)}{2} \qquad D = \prod_i D_i$$

$$D = \prod_i [\hat{u}_{ix}\hat{u}_{iy}] \prod_i [\imath d_i d_i']$$

$$f_j = \imath^j \frac{d_j + \imath d_j'}{2}$$

$$D = (-1)^{\hat{N}_\phi + \hat{N}_f}$$

$\hat{N}_f = \sum_i f_i^\dagger f_i$

Constraints on the physical states!

π - Flux Sectors

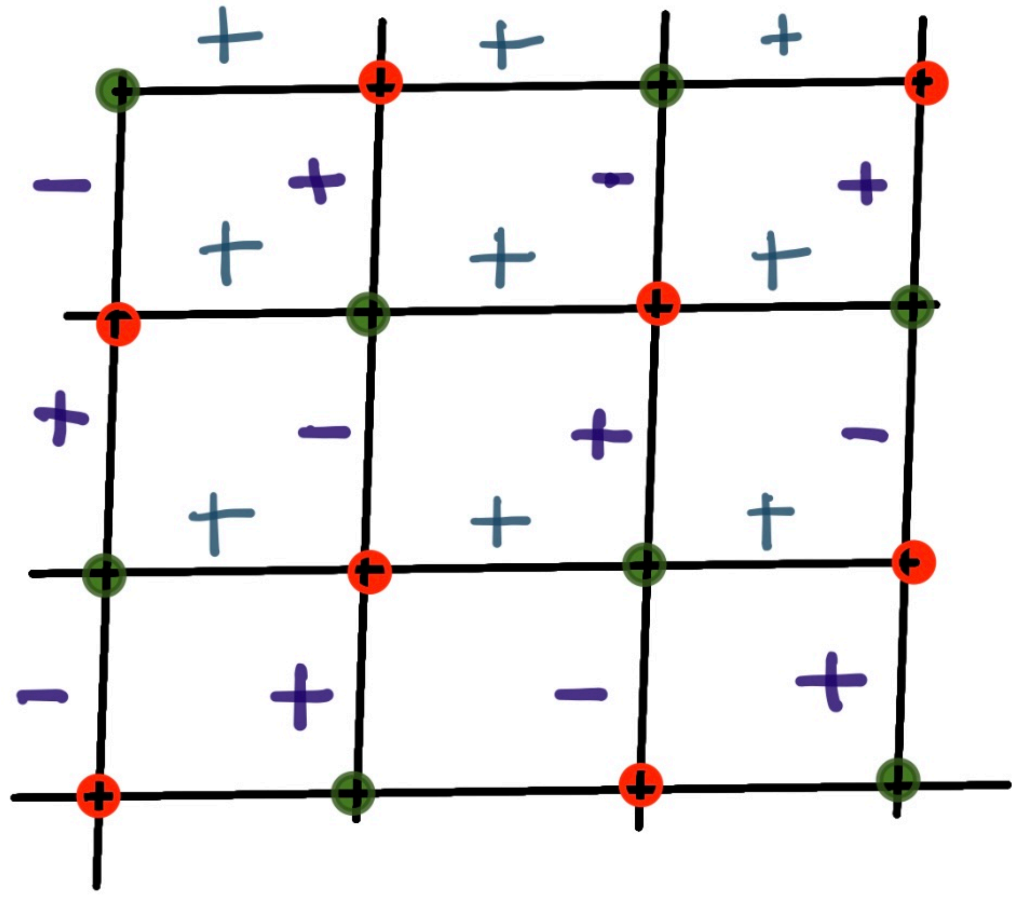
$J_5 = 0$

Lieb's Theorem

- Hopping on a square, bipartite lattice.
- Half-filled band of electrons.

$$\Downarrow$$

The ground state has π - flux per square plaquette.



Visons

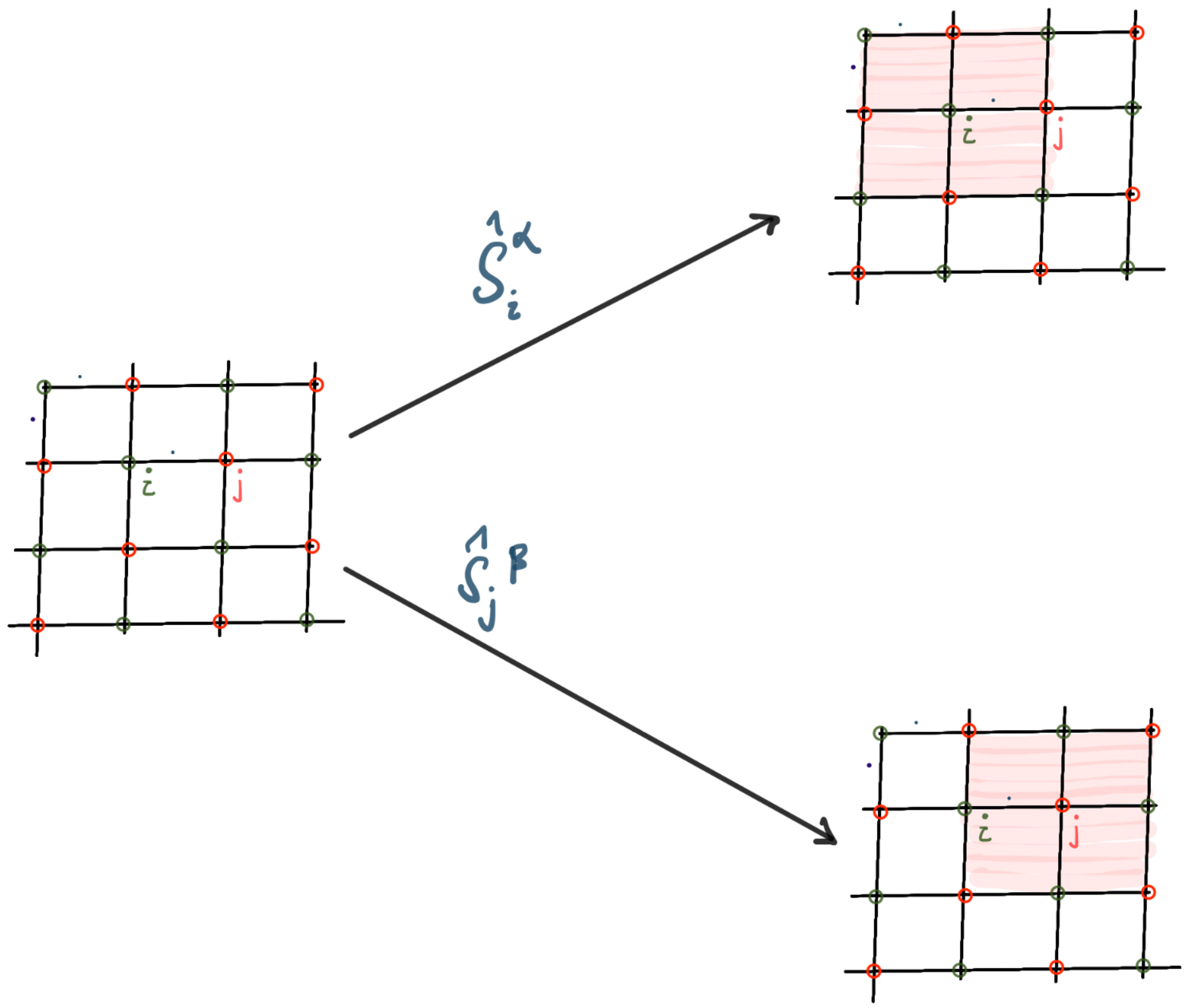
Square plaquettes with zero flux.

The visons are non-dynamical.

The vison excitation spectra is gapped.

$$\Delta_\nu \sim \left(\sqrt{|J_x J_y|} + \sqrt{|J'_x J'_y|}\right)$$


VISON EXCITATION


i
j
$\hat{S}_i^{\alpha}$
$\hat{S}_j^{\beta}$

Spin Correlation

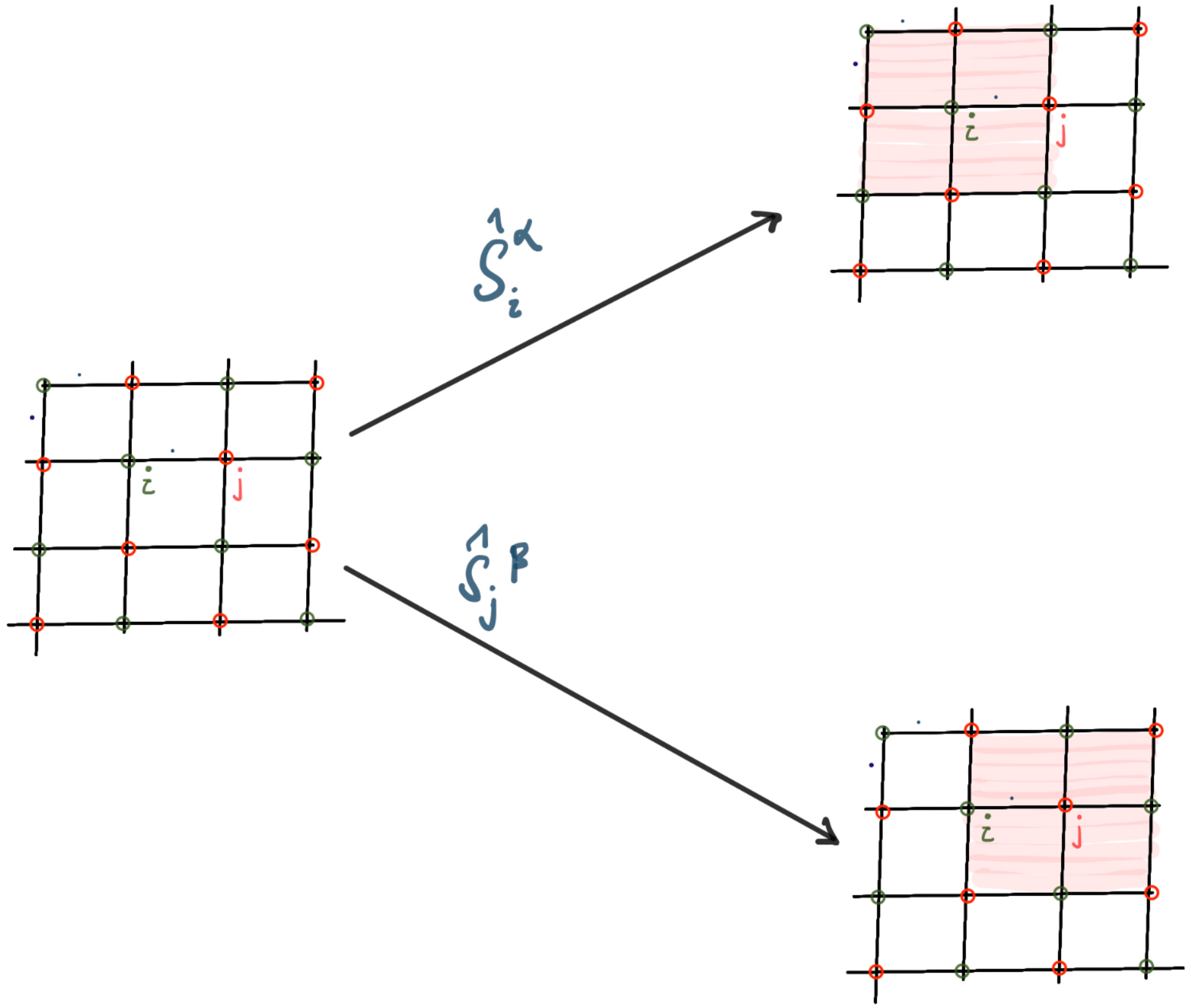
$$\left\langle \psi \left| S_i^{\alpha} S_j^{\beta} \right| \psi \right\rangle$$

Spin-3/2 operators can be written as bilinear forms of Majorana fermions.

- Short-ranged.
- Implies no magnetic ordering.

⇓

Spin liquid!



Spinon Spectrum

$$u_{ix} = 1, \;\; u_{iy} = (-1)^i$$

(Gauge fixing!)

$$H_0 = \sum_i \left(t_x f_u^\dagger f_{i+\hat{x}} + t_y (-1)^i f_i^\dagger f_{i+\hat{y}} - J_5 f_i^\dagger f_i - \Delta_x (-1)^i f_i^\dagger f_{i+\hat{x}}^\dagger - \Delta_y f_i^\dagger f_{i+\hat{y}}^\dagger + \text{h.c.} \right)$$

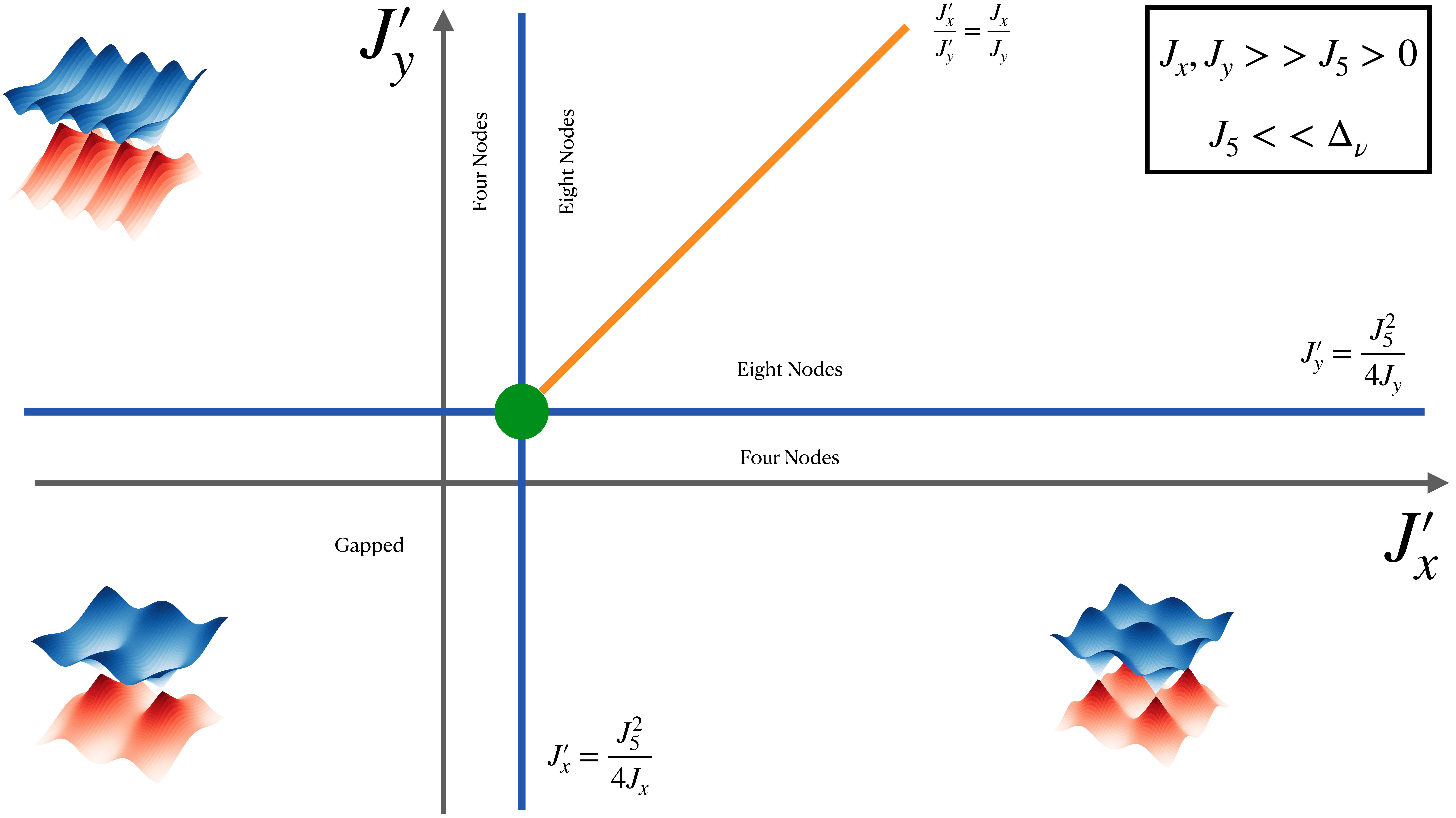
$t_\lambda = J_\lambda + J'_\lambda$ $\qquad$ $\Delta_\lambda = J_\lambda - J'_\lambda$

$$H = \begin{bmatrix} 2t_x \cos k_x - 2J_5 & 2\imath t_y \sin k_y & 2\imath \Delta_y \sin k_y & 2\Delta_x \cos k_x \\ -2\imath t_y \sin k_y & -2t_x \cos k_x - 2J_5 & -2\Delta_x \cos k_x & -2\imath \Delta_y \sin k_y \\ -2\imath \Delta_y \sin k_y & -2\Delta_x \cos k_x & -2t_x \cos k_x + 2J_5 & -2\imath \Delta_y \sin k_y \\ 2\Delta_x \cos k_x & 2\imath \Delta_y \sin k_y & 2\imath t_y \sin k_y & 2t_x \cos k_x + 2J_5 \end{bmatrix}$$

$$E_{\pm,\mathbf{k}} = 2\sqrt{J_5^2 + 2g_{+,\mathbf{k}} \pm 2\sqrt{g_{-,\mathbf{k}}^2 + J_5^2 g_{\mathbf{k}}}}$$

$$g_{\pm,\mathbf{k}} = (J_x^2 \pm J_x'^2)\cos^2 k_x + (J_y^2 \pm J_y'^2)\sin^2 k_y$$

$$g_{\mathbf{k}} = (J_x + J_x')^2 \cos^2 k_x + (J_y^2 + J_y')^2 \sin^2 k_y$$



J'_y
$\frac{J'_x}{J'_y} = \frac{J_x}{J_y}$
$J_x, J_y >> J_5 > 0$
$J_5 << \Delta_\nu$
Four Nodes
Eight Nodes
Eight Nodes
$J'_y = \frac{J_5^2}{4J_y}$
Four Nodes
J'_x
Gapped
$J'_x = \frac{J_5^2}{4J_x}$