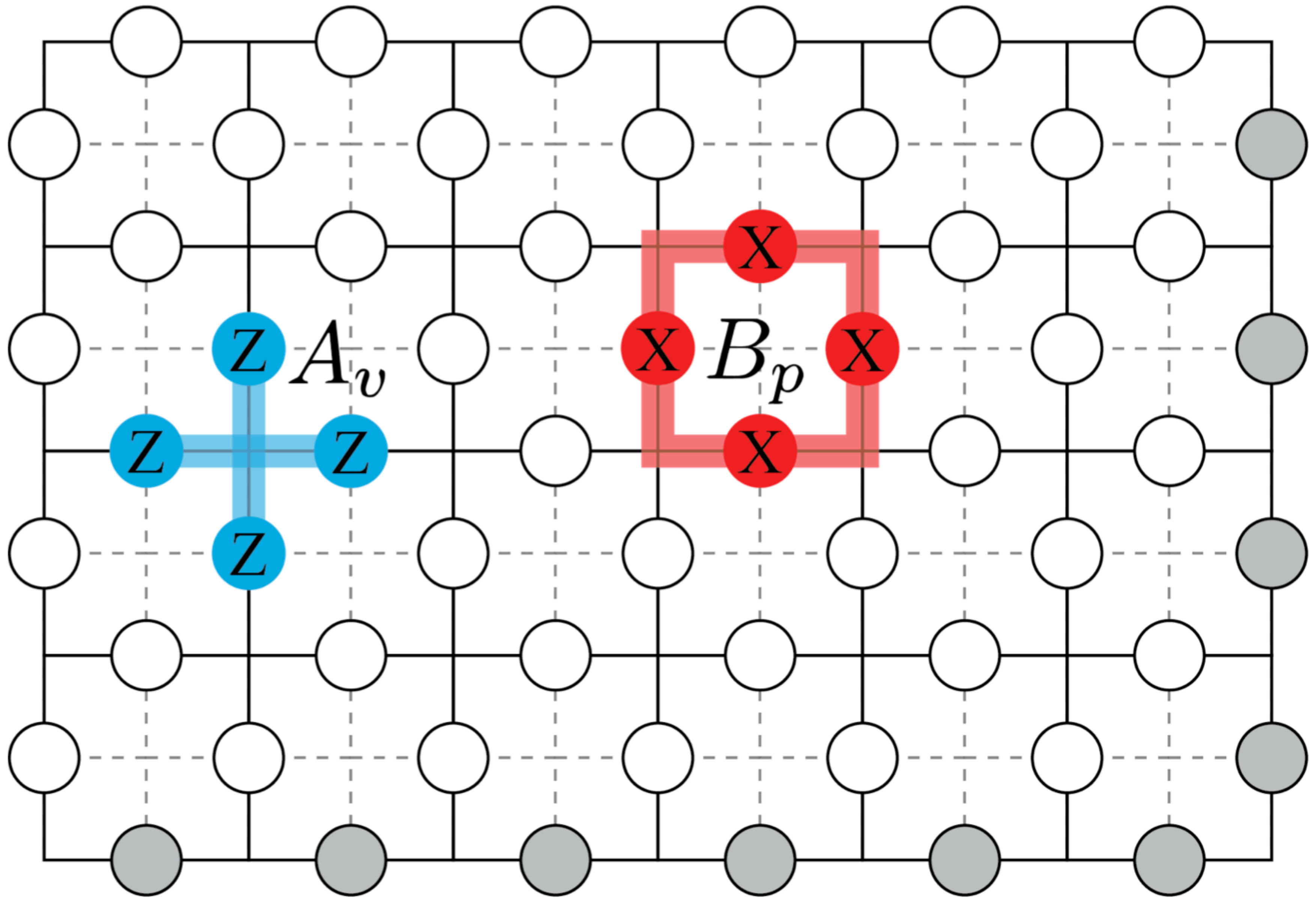
Quantum Error Correction

Surviving as a Quantum Computer in a Classical World

P471 Term-Paper Presentation

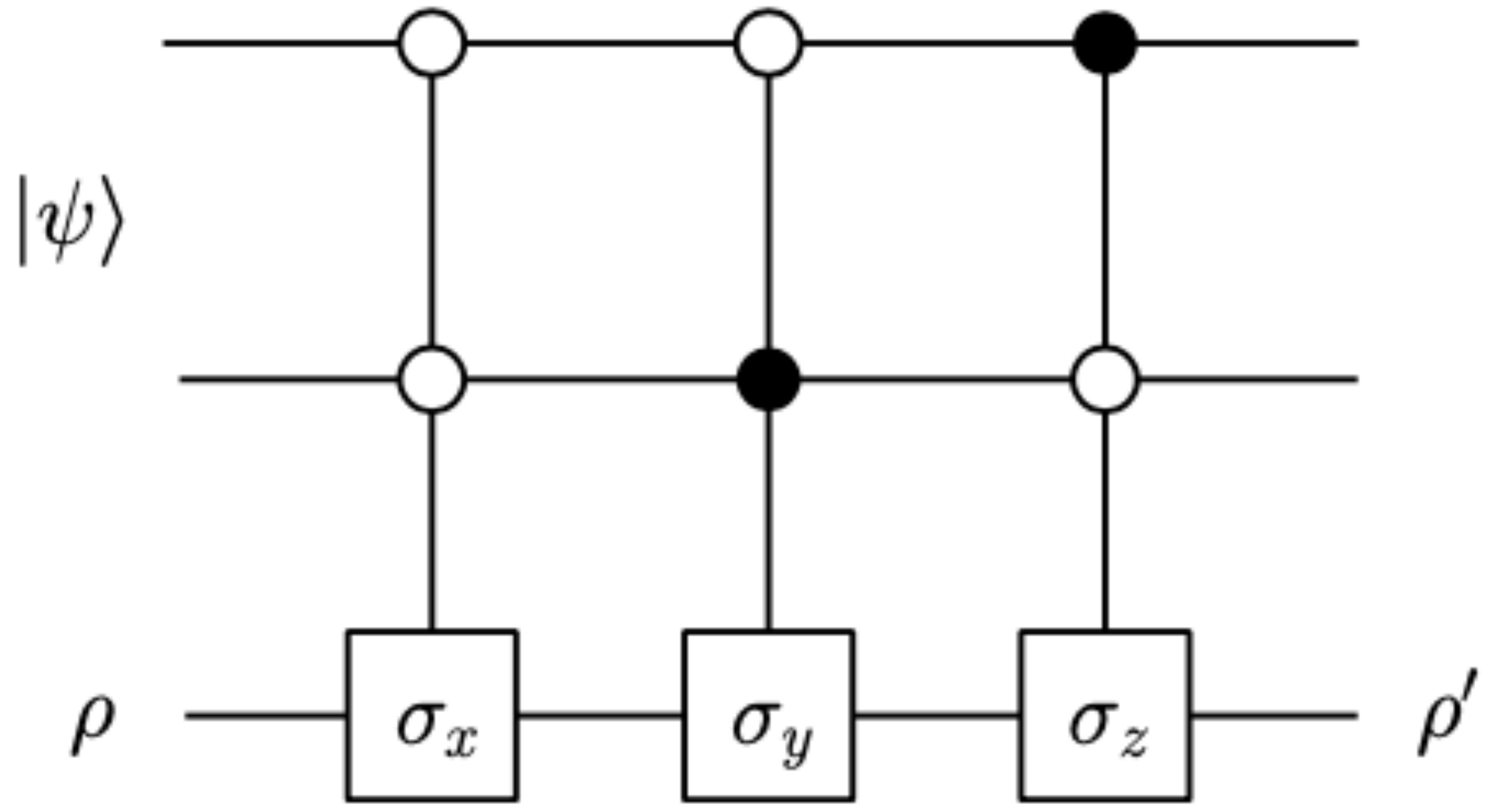
Deepak Kumar Sharma
Diptarko Choudhury
Jabed Umar
Sagar Prakash Barad
Sajag Kumar



Decoherence

Recall!

$$\rho = \frac{1}{2}(I + r \cdot \sigma) \longrightarrow \rho' = \frac{1}{2}(I + r' \cdot \sigma)$$



$$x' = \left(1 - 2|\beta|^2 + |\gamma|^2\right) x$$
$$y' = \left(1 - 2|\gamma|^2 + |\alpha|^2\right) y$$
$$z' = \left(1 - 2|\alpha|^2 + |\beta|^2\right) z$$

$$|\Psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$$

Different values of $\alpha, \beta, \gamma, \delta$ leads to different kinds of noisy channels.

Classical Error Correction

The general errors are bit flips.

$$0 \rightarrow 1$$
$$1 \rightarrow 0$$

Repetition Code

The naive way is to add redundancy.

$$0 \rightarrow 000$$

Apply majority voting rule.
Reduces the chance of errors.

Classical Linear Code

Encode ***k*** bits in ***n*** bits using a generator matrix **G.**

Parity matrix H acting on codewords vanishes.

Suppose an error ***e*** occurs such that

$$y' \rightarrow y + e \implies Hy' = He$$

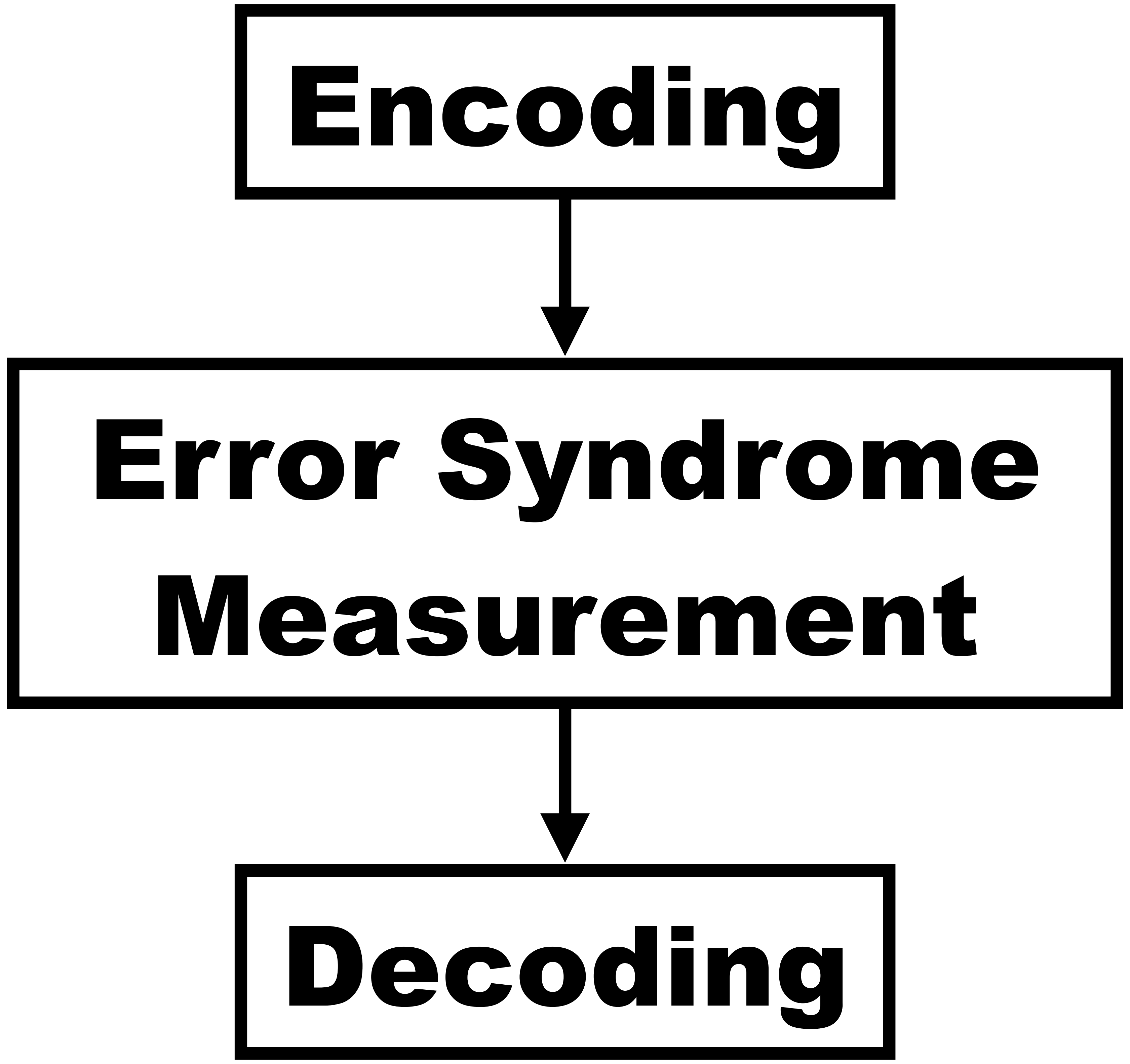
Classical errors can be detected and corrected iff all the He's are distinct.

$$H = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

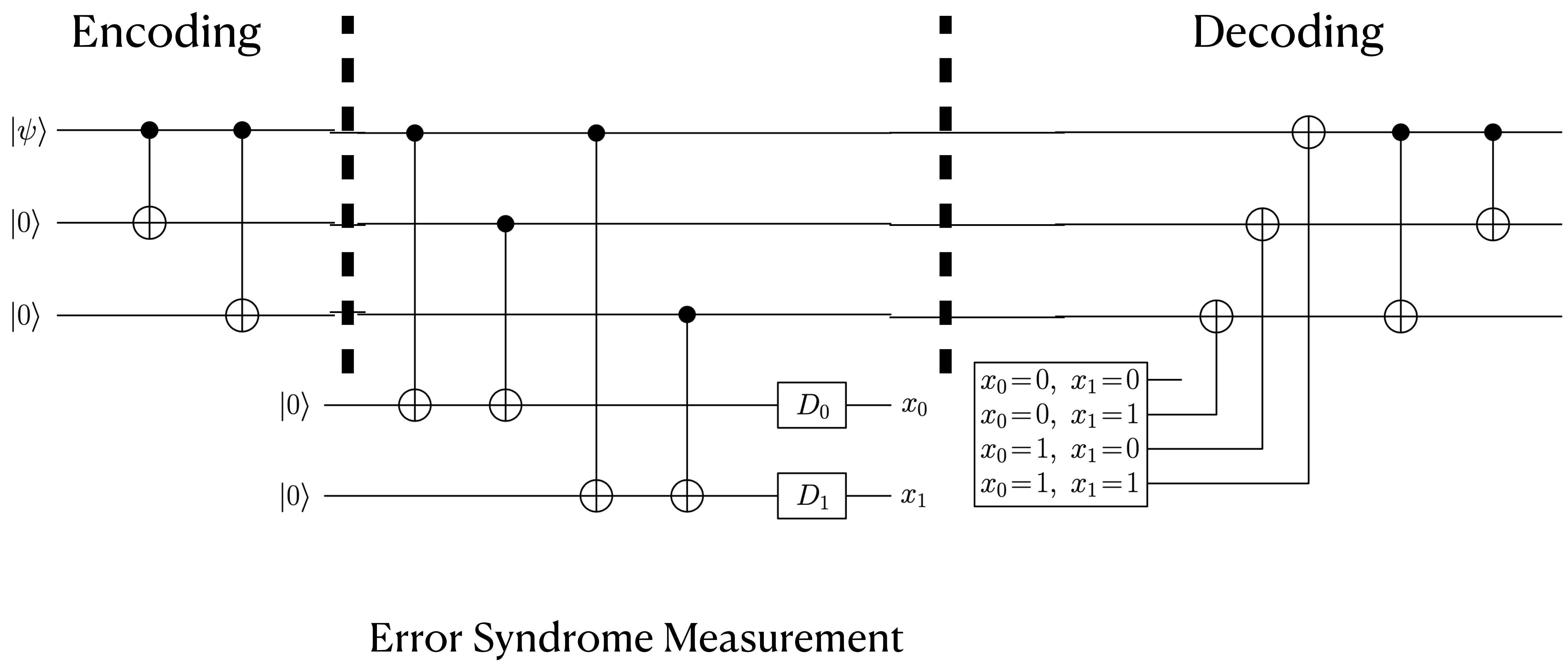
$$He_1 = (11)^T, He_2 = (10)^T, He_3 = (01)^T$$

So single bit flip errors can be detected and corrected for the 3 bit repetition code.

Quantum Error Correction



Encoded state sent by Alice to Bob through a noisy quantum channel.



	$\vert 000\rangle$	$\vert 100\rangle$	$\vert 010\rangle$	$\vert 001\rangle$
D_0	0	1	1	0
D_1	0	1	0	1

The Stabiliser Formalism

$$\mathcal{G}_n = \{\pm 1, \pm \iota\} \otimes \{I, X, Y, Z\}^{\otimes n}$$

Stabiliser group

$$\hat{O}|\Psi\rangle = |\Psi\rangle$$

Stabilised vector space

$$V_S$$

Generators of the stabiliser group

$$\langle g_1,\ g_2,\ \ldots,\ g_n \rangle$$

Logical states

$$\langle g_1, \ldots, g_n, (-1)^{x_1} \bar{Z}_1, \ldots, (-1)^{x_k} \bar{Z}_k \rangle.$$

Syndrome measurement of the generators

$$\beta_1, \beta_2, \beta_3, \ldots, \beta_4$$

The Gottesman-Knill Theorem

Fault-Tolerant Quantum Computation

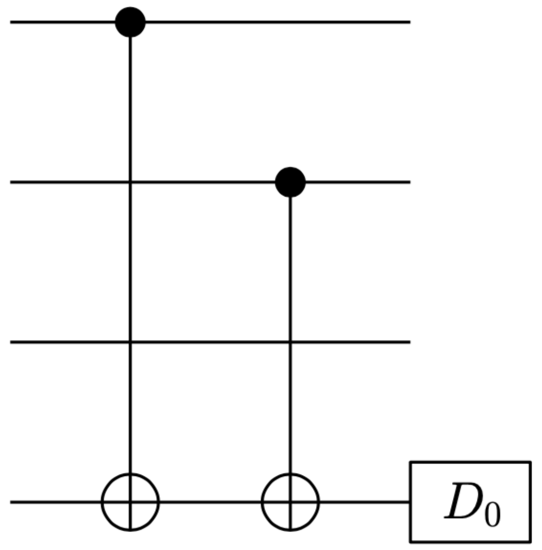
Gates are not ideal!

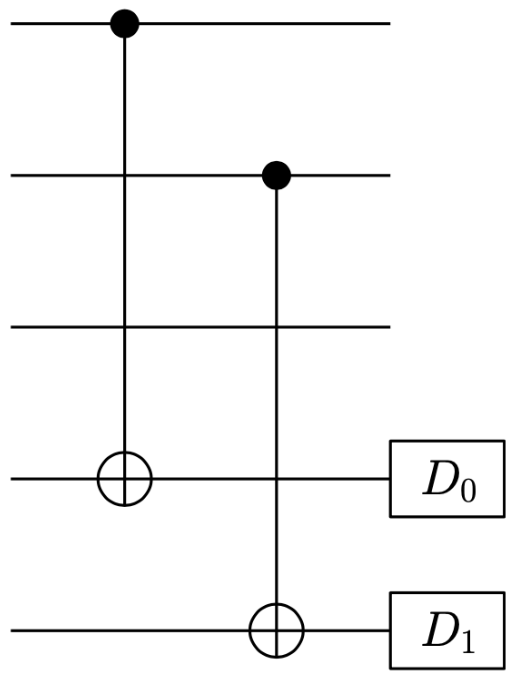
There are many sources of errors.
Unitary gates, measurement, interaction with the environment.
And they propagate!

Yet,

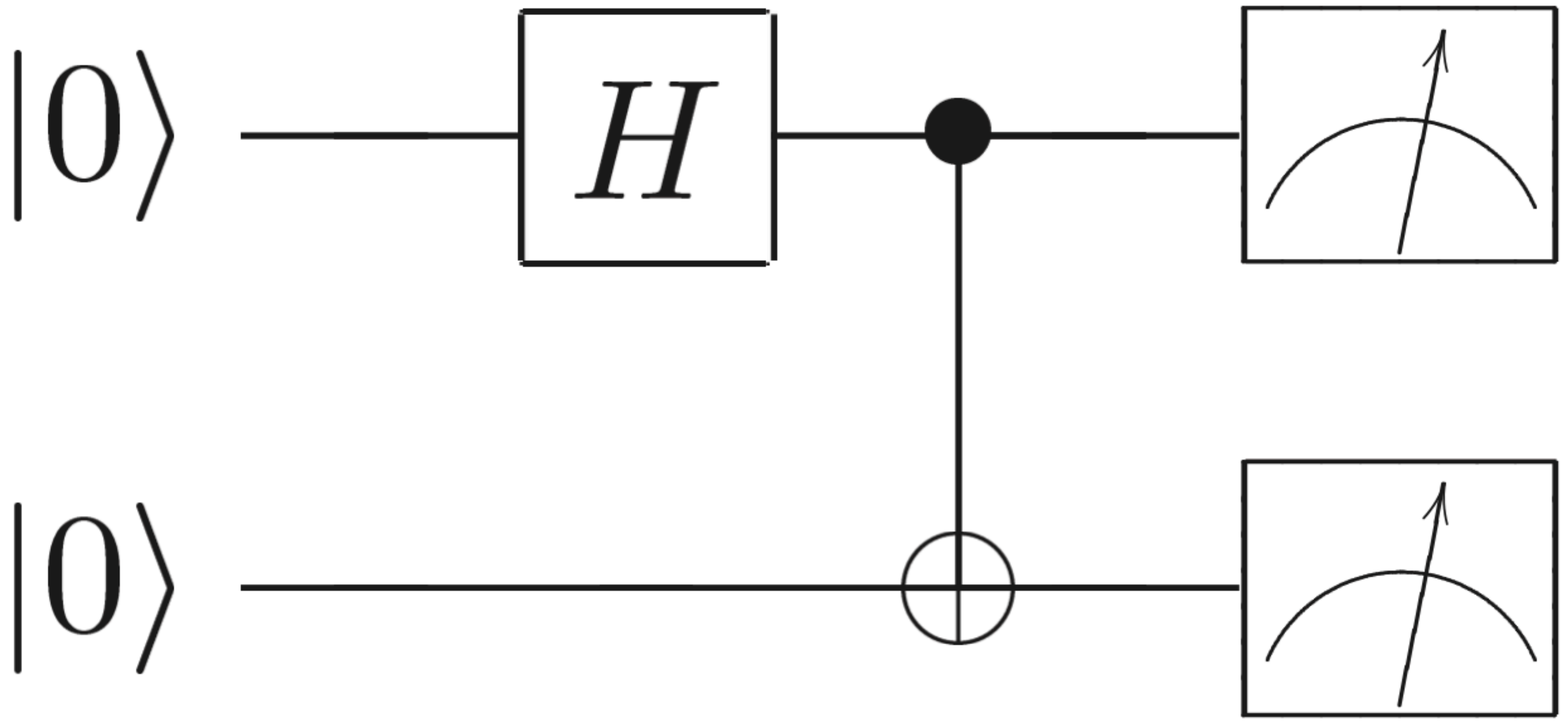
We can perform **arbitrarily long** quantum computation, in principle.

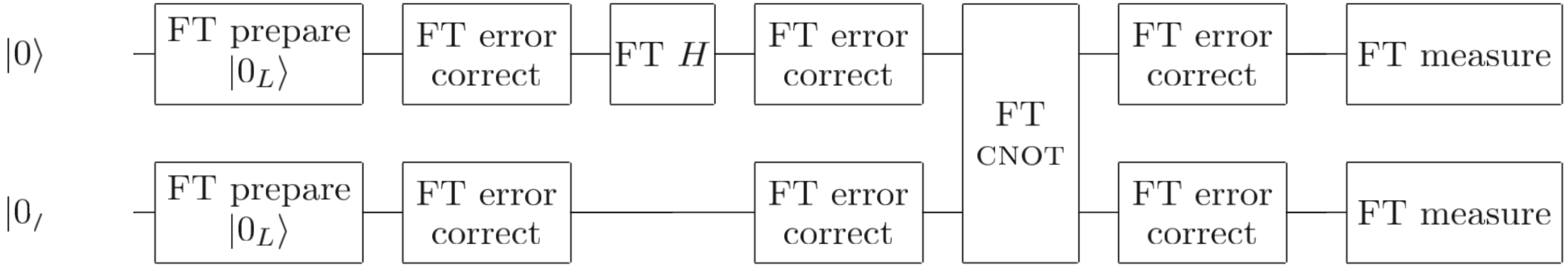
Avoid error propagation.





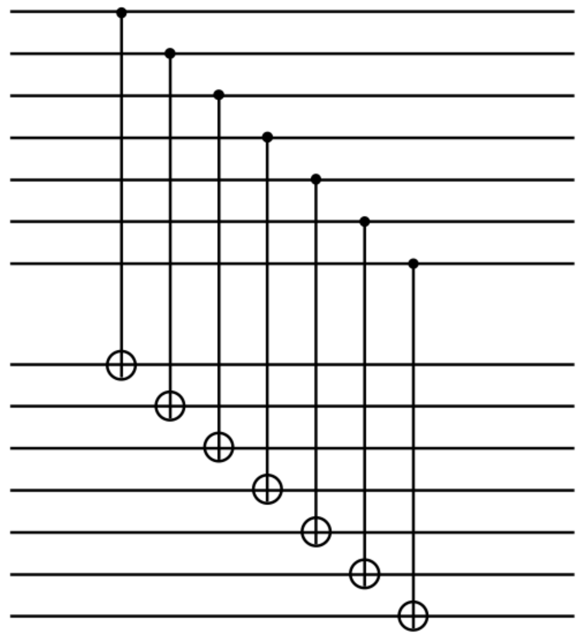
Corrects **t** errors then failure probability should be $\mathcal{O}(\epsilon^{t+1})$



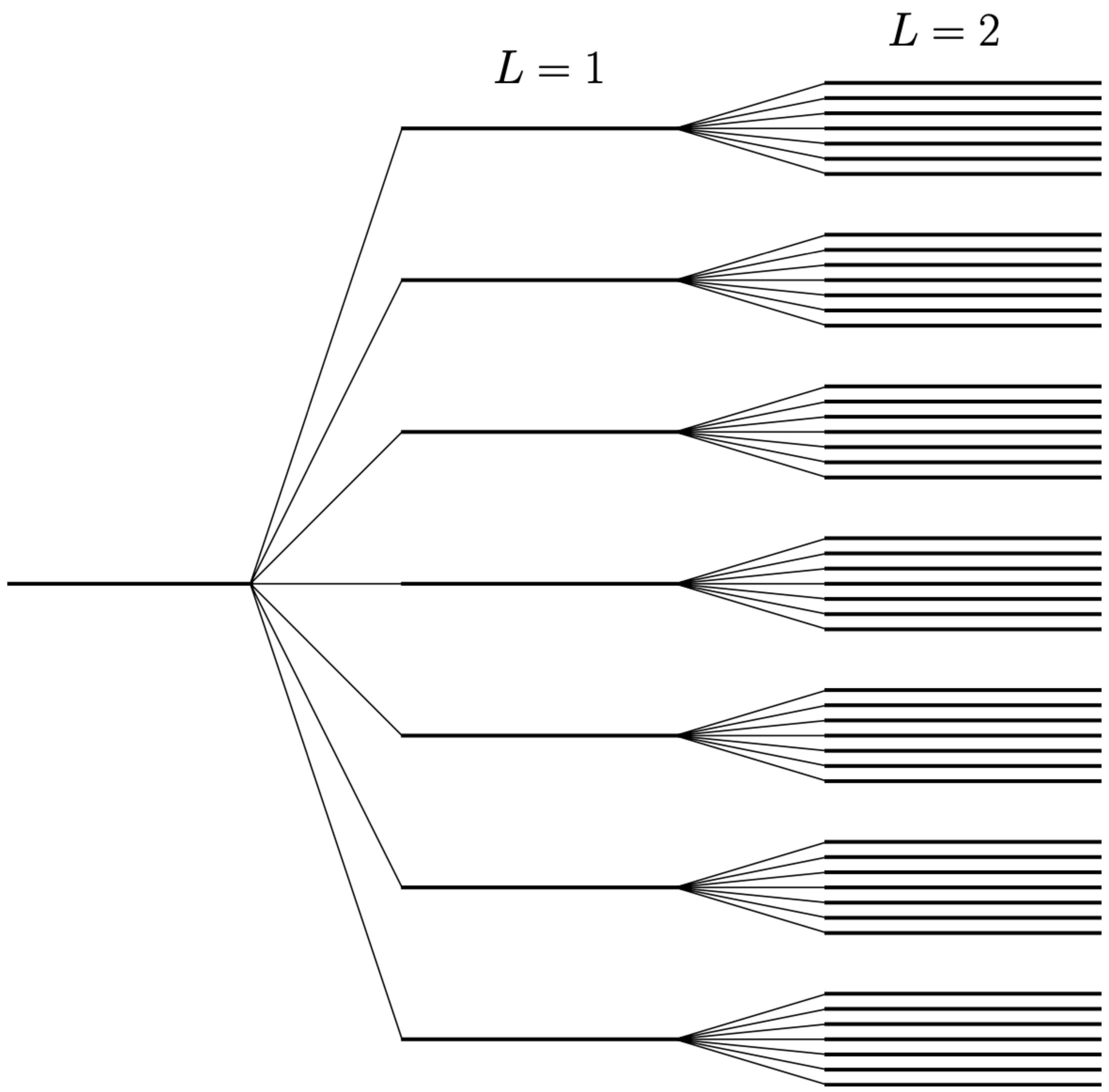


Need a fault-tolerant prescription for preparation, measurement and error correction.

Need fault tolerant gates!



The Threshold Theorem



$$p_L \approx \frac{(\alpha^2\epsilon)^{2^L}}{\alpha^2}$$

α - number of sites where error can happen.
ϵ - error probability rate.

$$p_L \leq \frac{\epsilon_0}{T}$$

T - number of logical quantum gates.
ϵ_0 - accuracy.

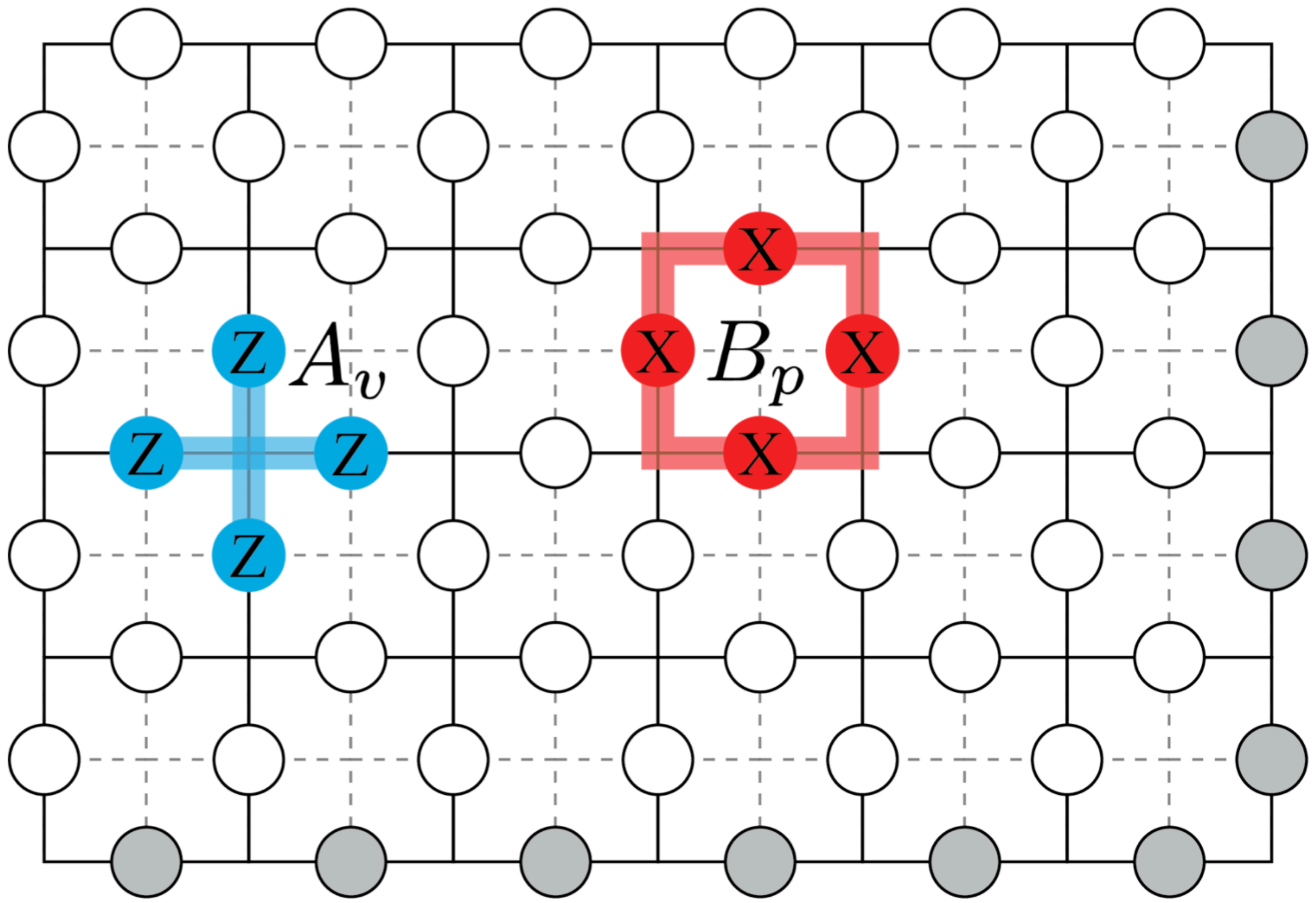
$$L > \bar{L} \sim \log\left[\frac{\log\left(T/(\alpha^2\epsilon_0)\right)}{\log\left(1/(\alpha^2\epsilon_0)\right)}\right]$$

$$n \sim \left[\frac{\log\left(T/(\alpha^2\epsilon_0)\right)}{\log\left(1/(\alpha^2\epsilon_0)\right)}\right]^{\log n}$$

n - number of required qubits.

The Toric Code

$$H = -\sum_v A_v - \sum_p B_p$$



$$A_v = \prod_{j \in \mathrm{star}(v)} Z_j \quad \Bigg| \quad B_p = \prod_{j \in \mathrm{bdy}(p)} X_j$$

$$[A_v, B_p] = 0$$

$$A_v^2 = B_p^2 = 1$$

So the eigenvalues are +**1** or **-1**.

The ground state of H is the simultaneous eigenstate of the star and plaquette operators with eigenvalues +**1**.

Ground State

Dimension of the ground state manifold

(N - 1) constraints from star operator.

(N-1) constraints from plaquette operator.

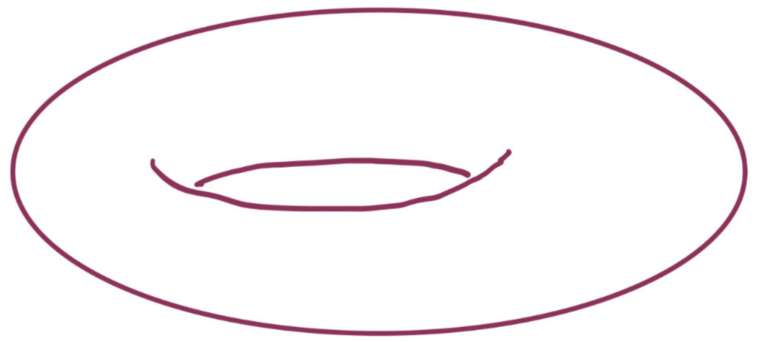
↓

(2N-2) total constraints.

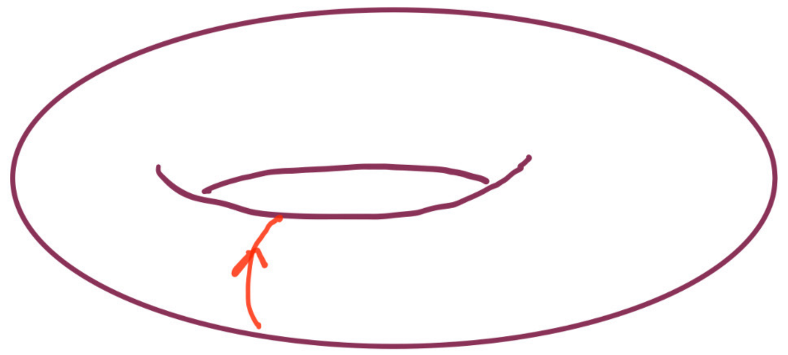
↓

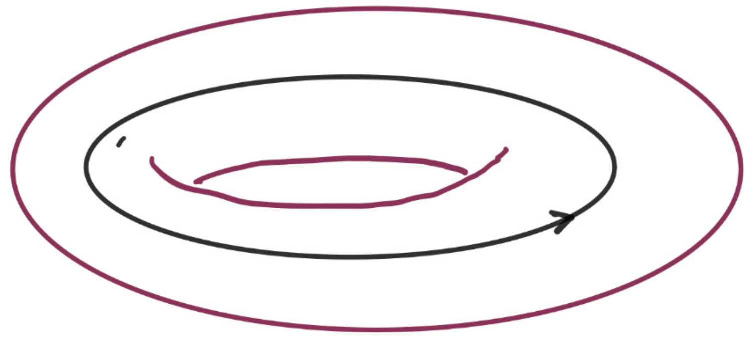
Only 2 independent degrees of freedom.

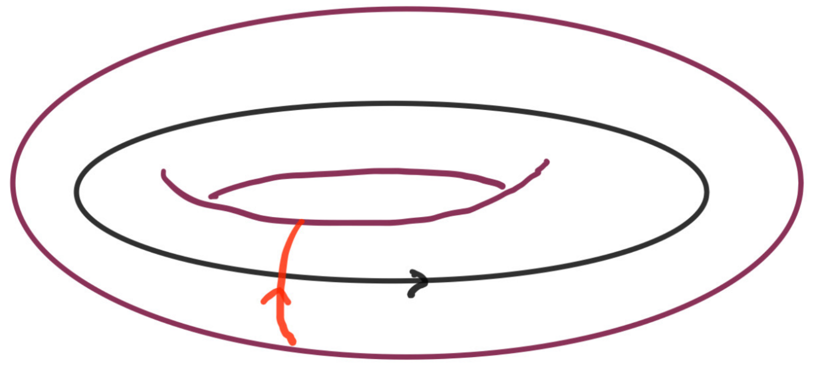
Trivial Ground State



Non-Trivial Ground State

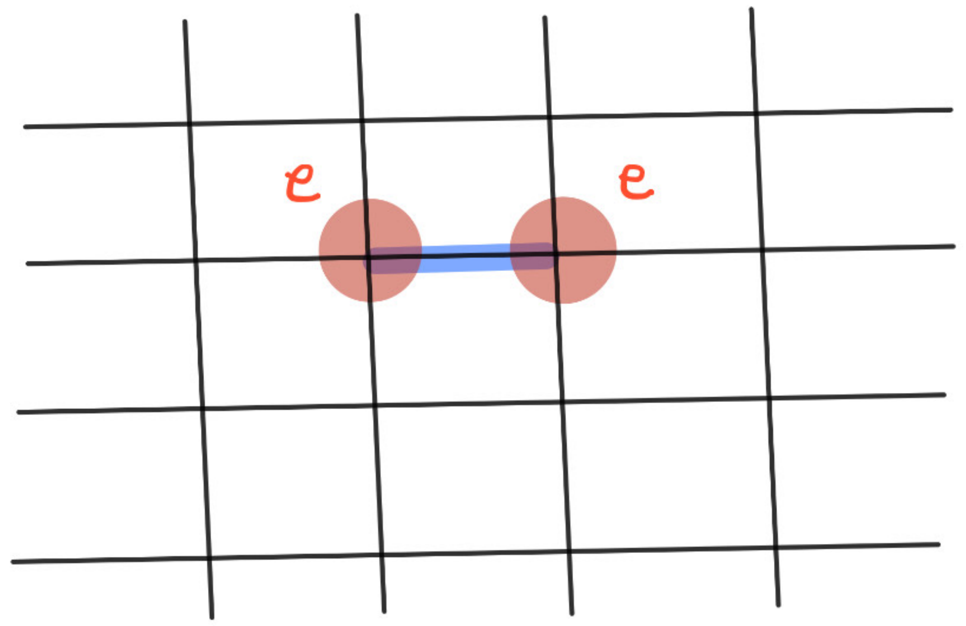


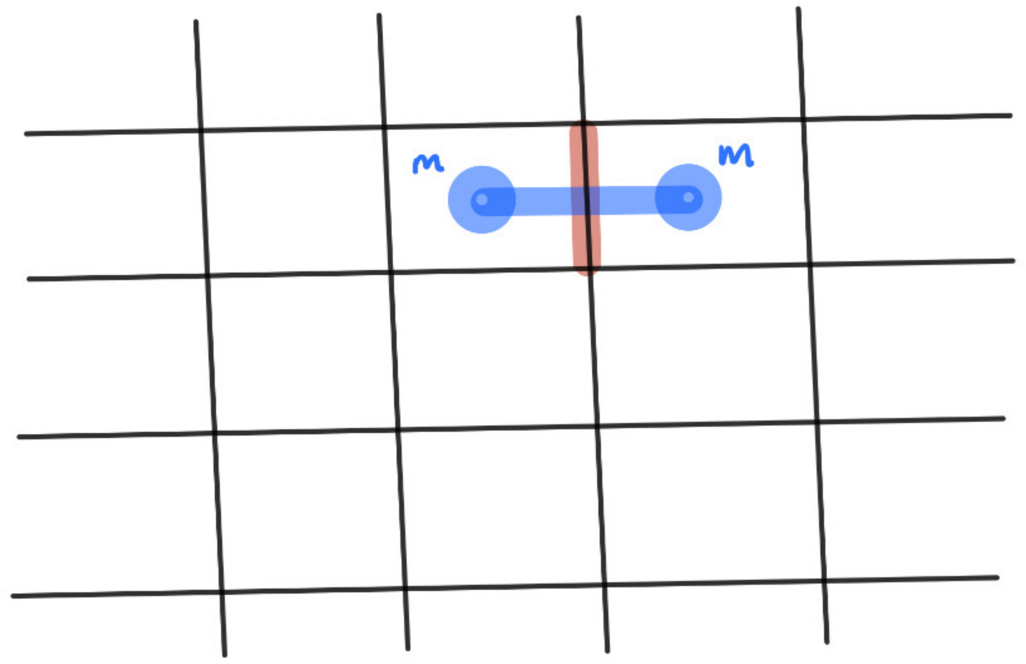


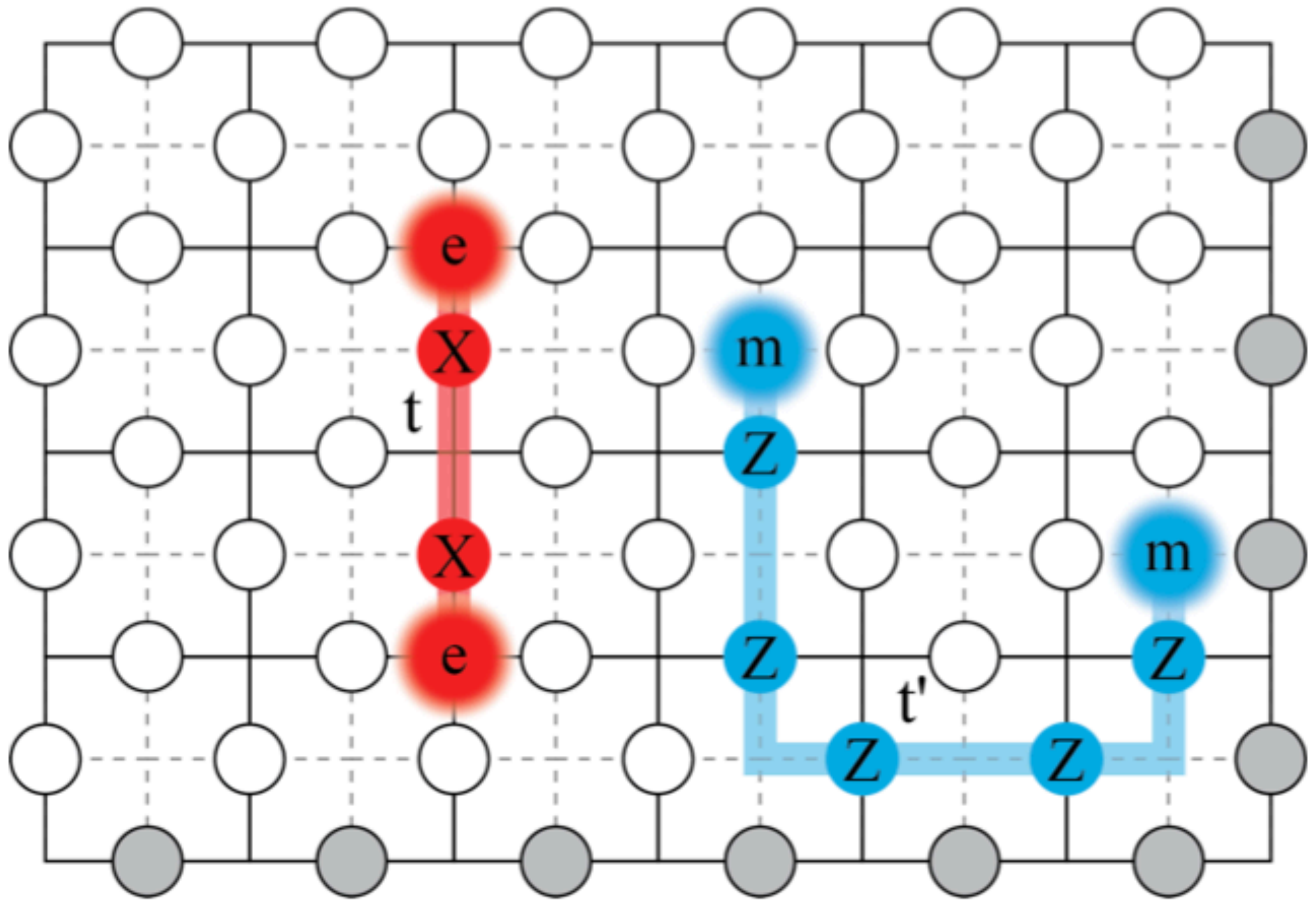


The ground state manifold is four dimensional.

Excited States







The Hamiltonian is gapped

There is an energy gap between the ground state and the excited states.

$$\Delta E \geq 2$$

The difference in the eigenvalues of the star/plaquette operators is $\mathbf{2}$.

Toric code is a stabiliser code!

Generators of the stabiliser group $\{A_v, B_p\}$

The vector space stabilised is spanned by the ground states.

The excitations form the set of correctable errors.

The following operators can perform computation

$$L_{xe} = \prod_{j \in \text{all}(x)} X_j \qquad L_{xm} = \prod_{j \in \text{all}(x)} Z_j \qquad L_{ze} = \prod_{j \in \text{all}(z)} X_j \qquad L_{zm} = \prod_{j \in \text{all}(x)} Z_j$$

Because there was a gap between the ground and the excited states,
the toric code is fault tolerant.

Thank you!