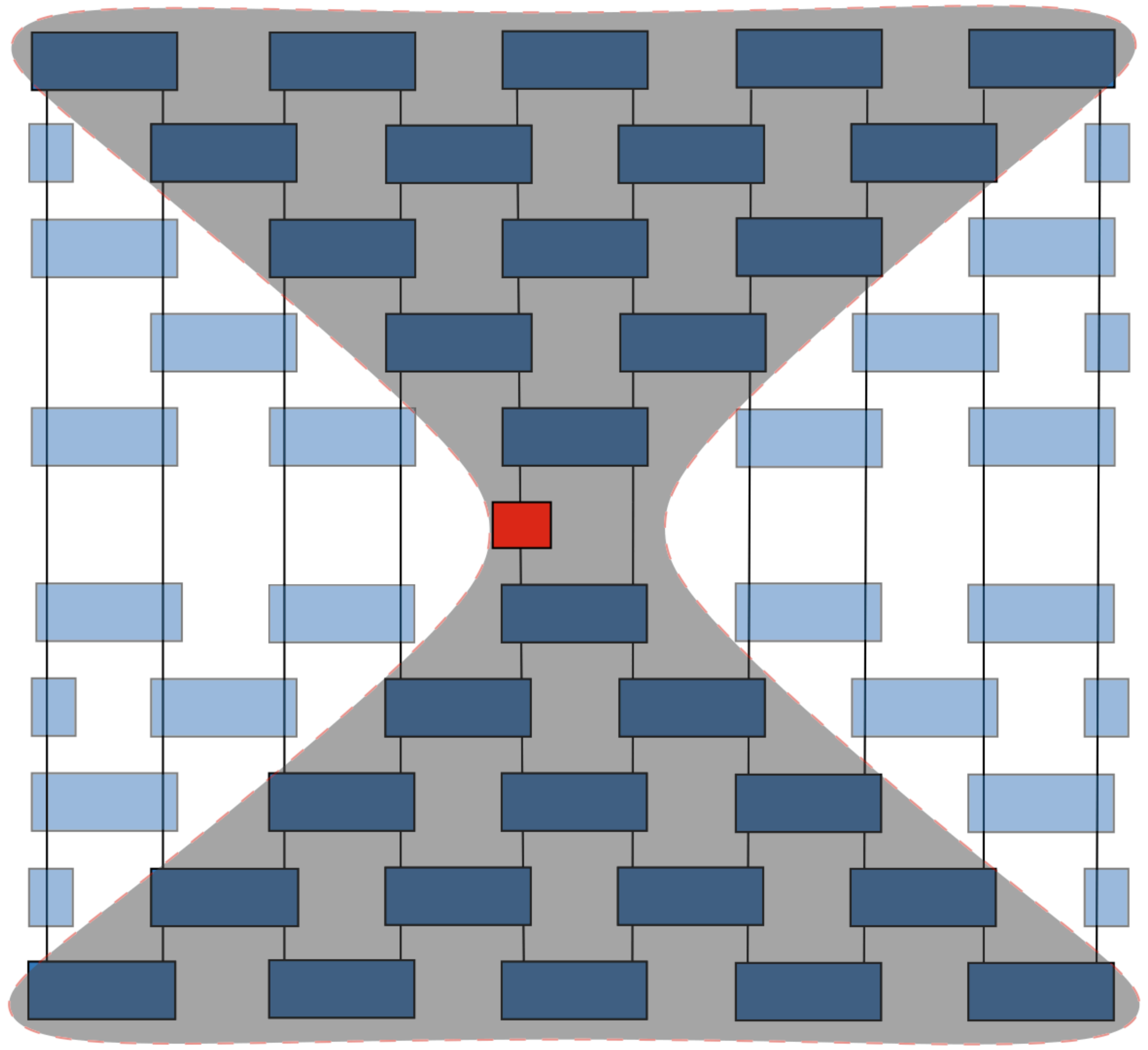
Probing Scrambling Dynamics Through Out-of-Time Order Correlators

P452 Term Paper Presentation



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Quantum Many-Body Systems

↓

Thermalize

Expectation value of local operators equals the ensemble expectation value.

↓

Information has flown to non-local degrees of freedom.

Scrambling

Operator Spreading

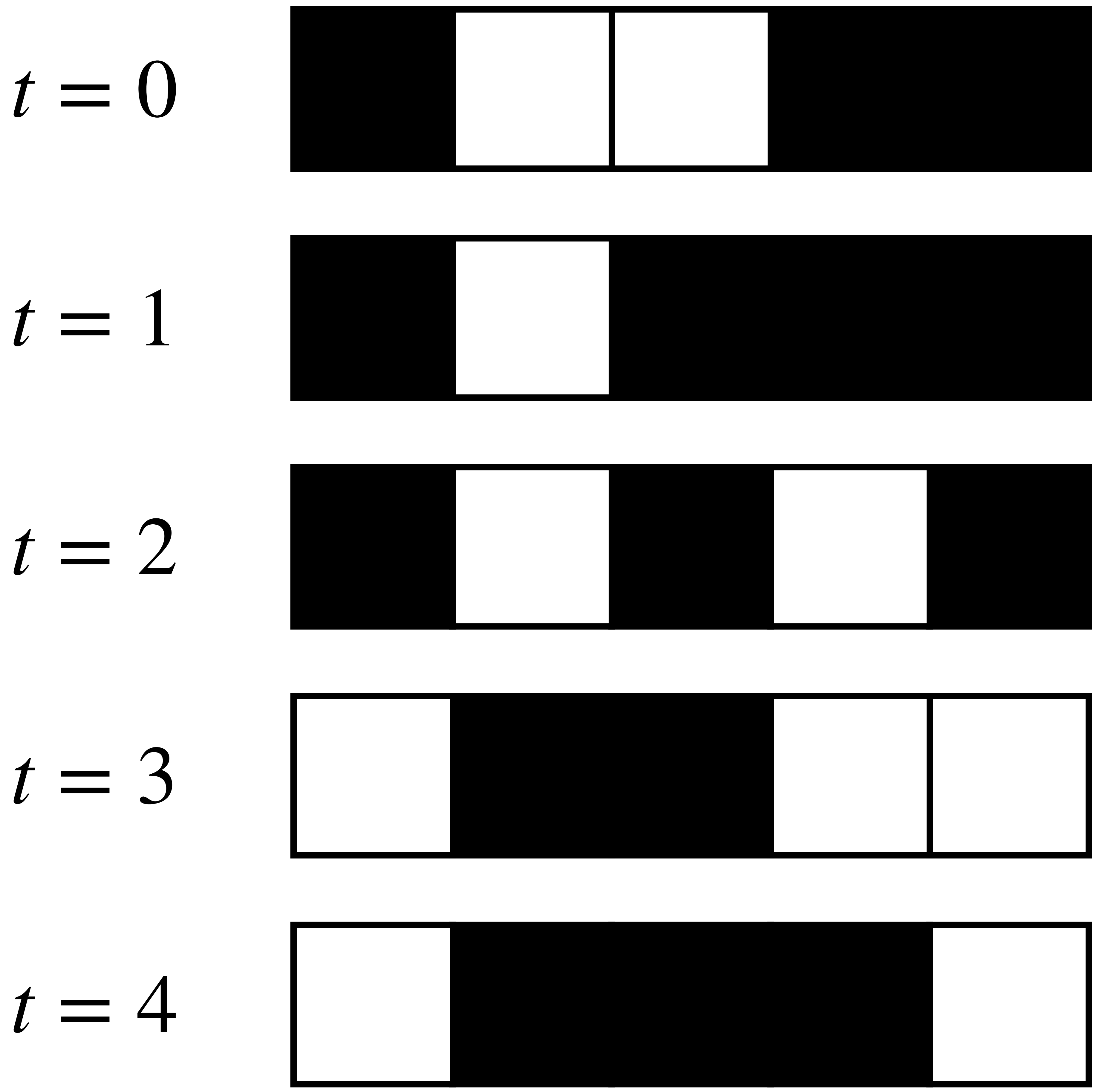
$$W(t) = \underbrace{e^{iHt}}_{\text{backward}} \underbrace{W}_{\text{perturbation}} \underbrace{e^{-iHt}}_{\text{forward}}$$

↓

Measured by the out-of-time order correlator.

$$OTOC = \langle W(x,t)V(0,0)W(x,t)V(0,0)\rangle$$

Kauffman Cellular Automaton



$$\sigma(r, t+1) = f_{r,t}[\sigma(r-K, t), \ldots, \sigma(r, t), \ldots, \sigma(r+K, t)]$$

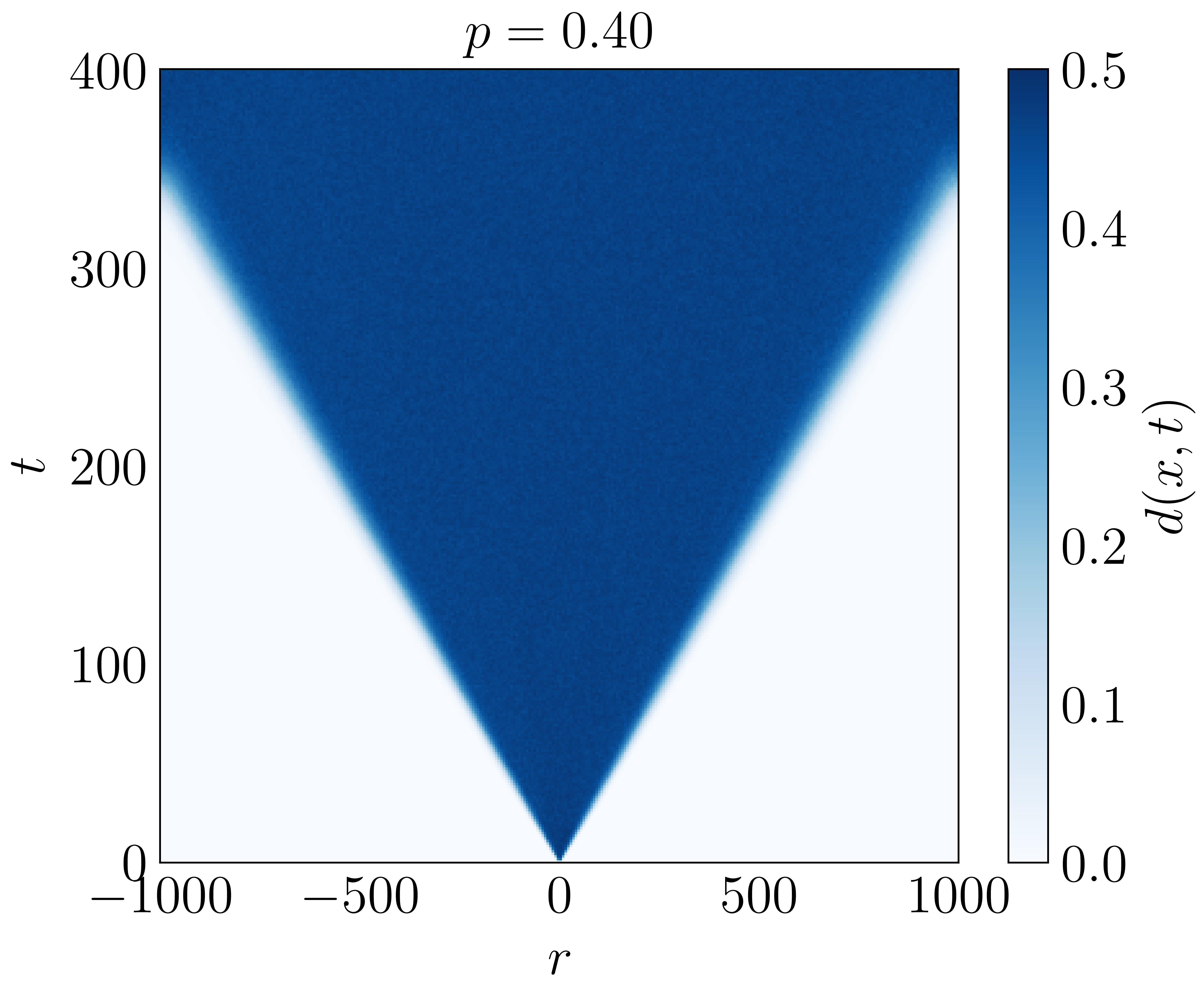
Set of local rules

$$f_{r,t} = \begin{cases} +1 \text{ with probability p} \\ -1 \text{ with probability 1-p} \end{cases}$$

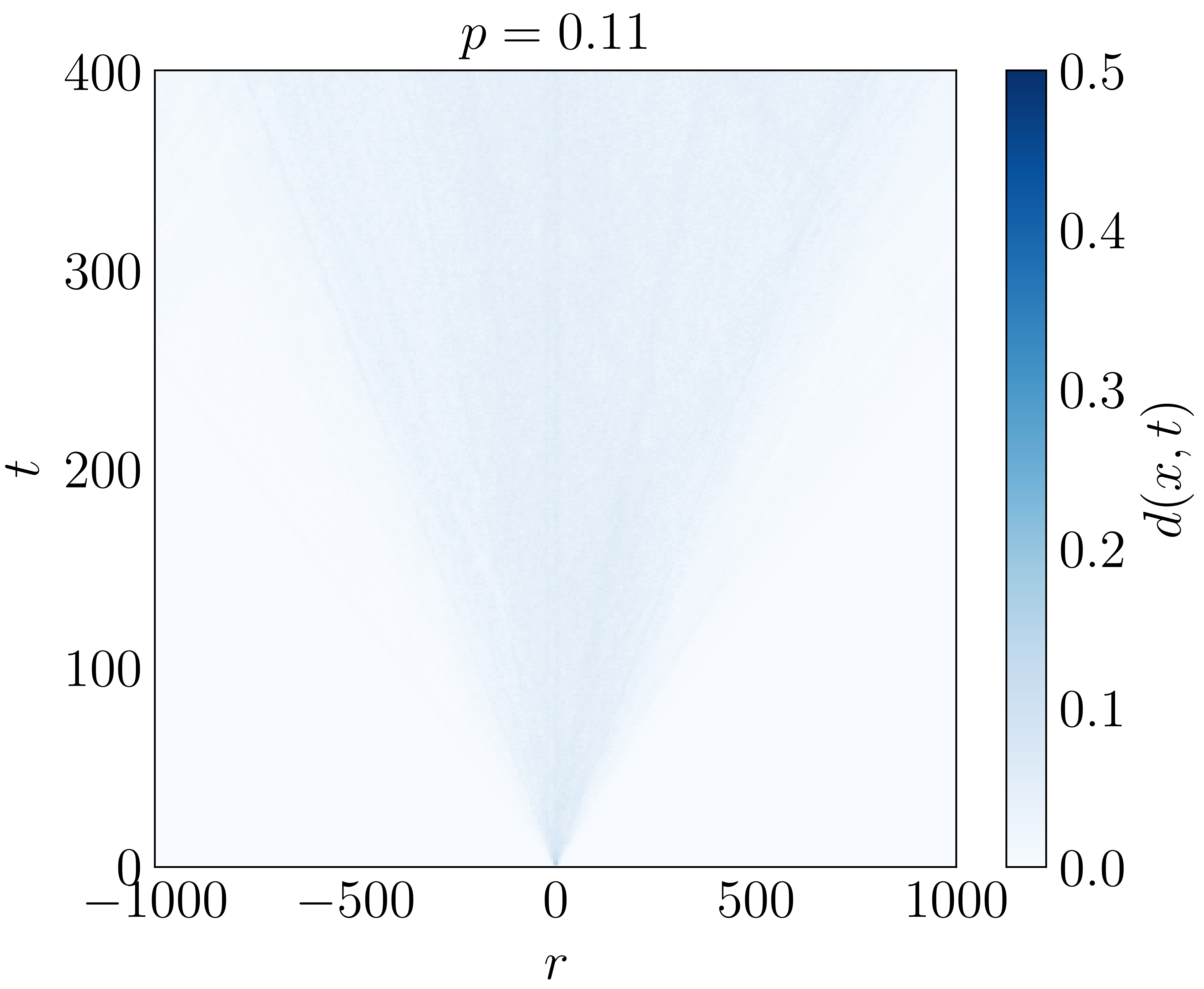
Decorrelator

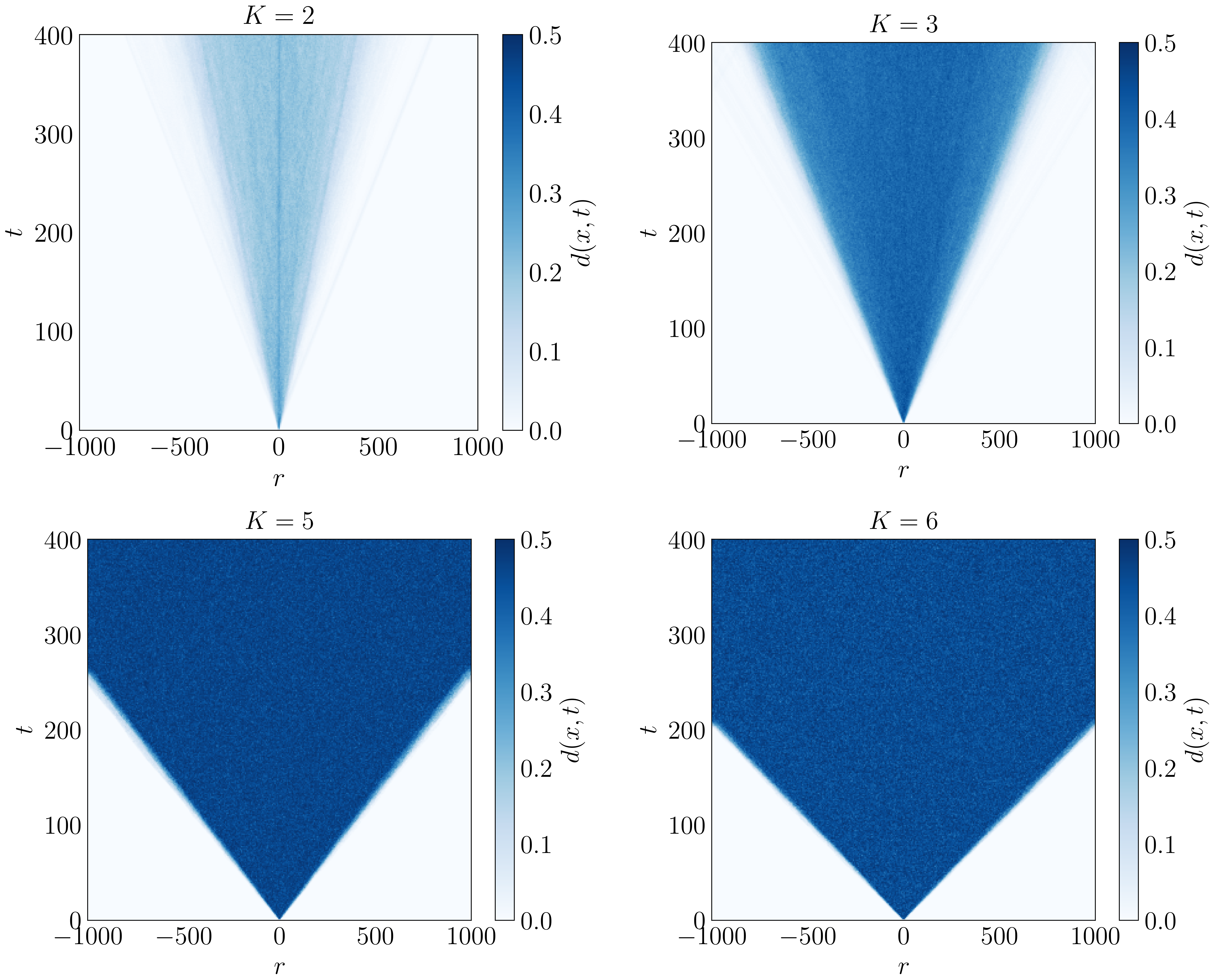
$$d(r, t) = \frac{1}{2}[1 - \langle \sigma^A(r, t)\sigma^B(r, t) \rangle_p]$$

Chaotic Phase

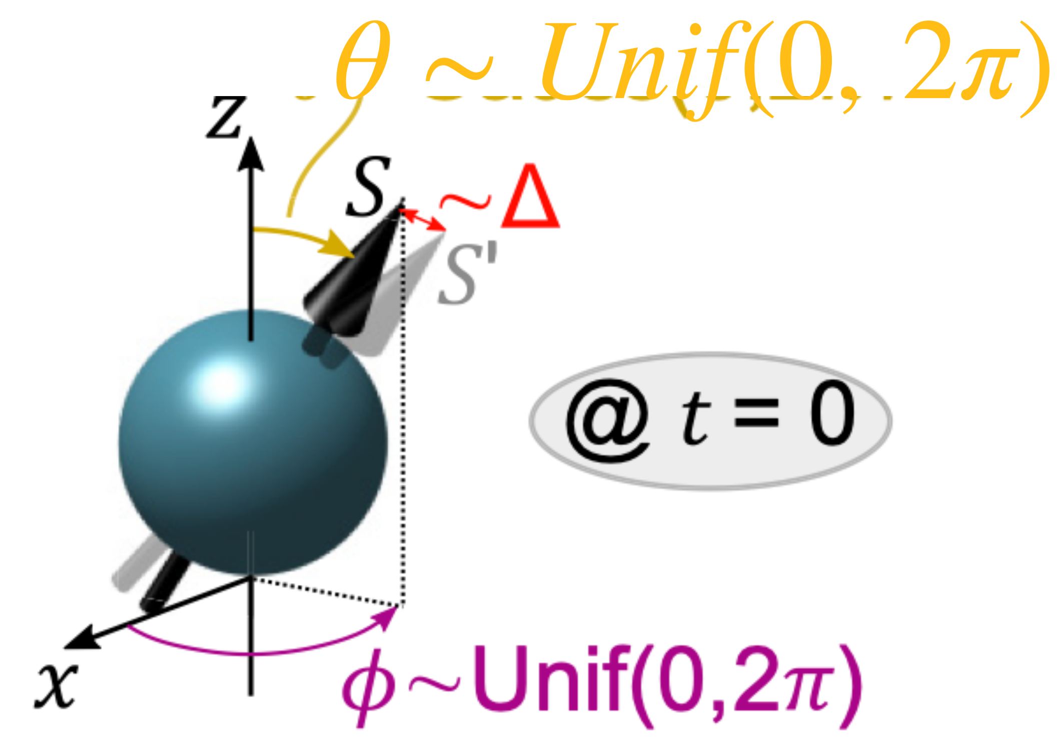


Frozen Phase




$K = 2$
$K = 3$
$K = 5$
$K = 6$
t
r
$d(x, t)$
0
100
200
300
400
−1000
−500
0
500
1000
0.0
0.1
0.2
0.3
0.4
0.5

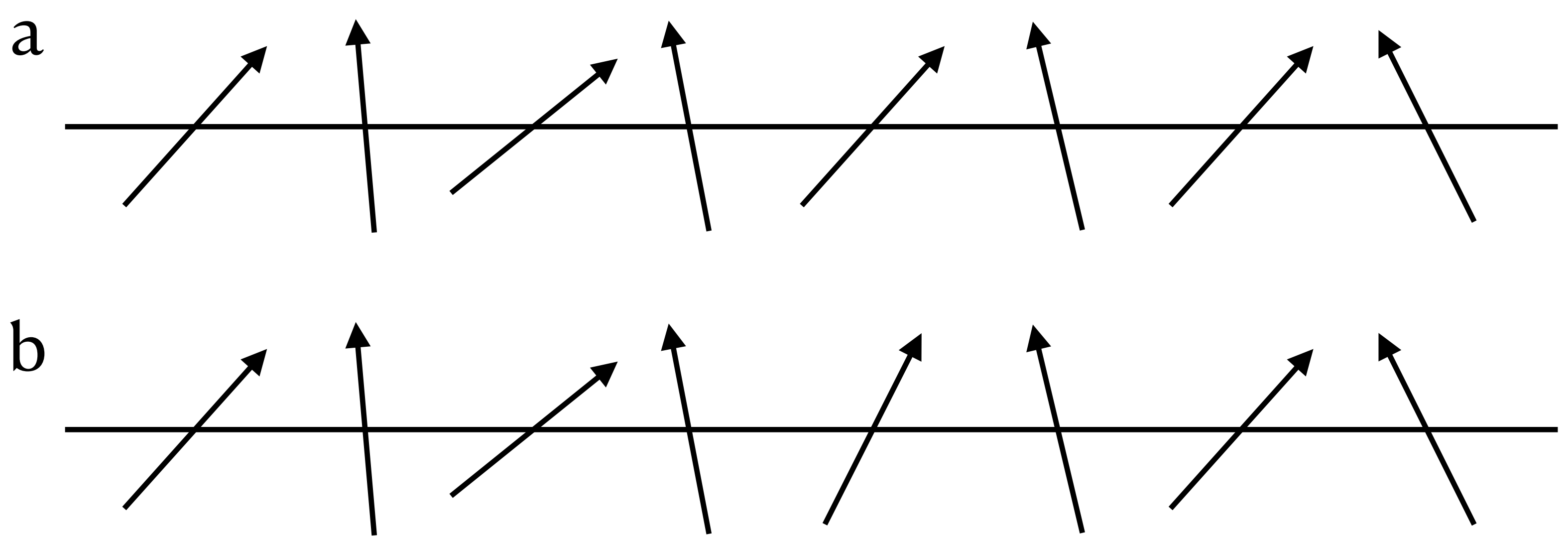
Classical Spins on a 1D Lattice



$$\{S_i^\alpha, S_{i'}^\beta\} = \delta_{ii'}\varepsilon^{\alpha\beta\gamma}S_i^\gamma$$

Classical Equation of Motion

$$\partial_t S_i = \{S_i, H\}$$



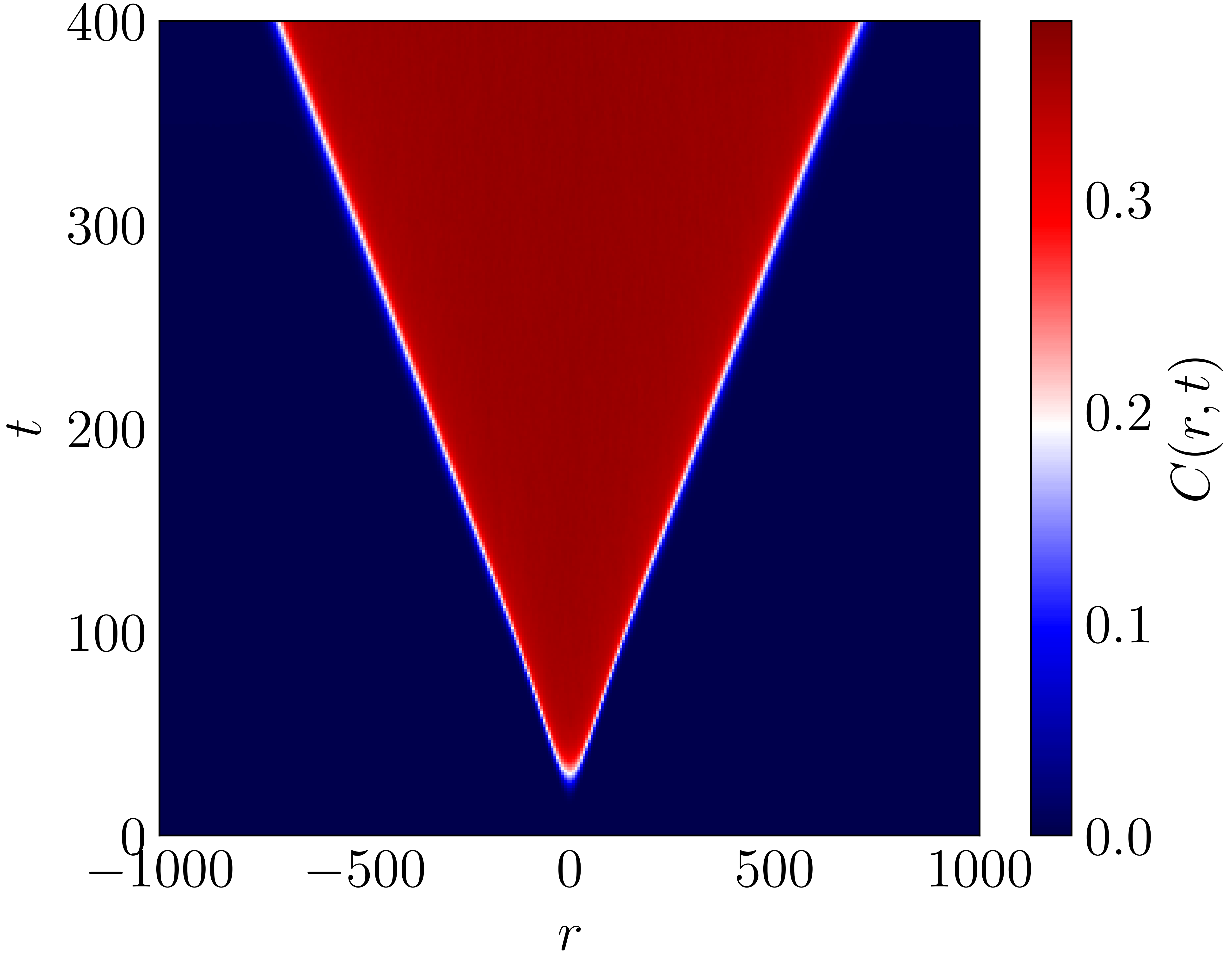
$$D(r,t) = \frac{1}{2}(1 - \langle \mathbf{S}_r^a(t) \cdot \mathbf{S}_r^b(t) \rangle)$$

The Classical Ising Model

$$\mathscr{H} = -J \sum_{i=0}^{N-1} \mathbf{S_i} \cdot \mathbf{S_{i+1}}$$

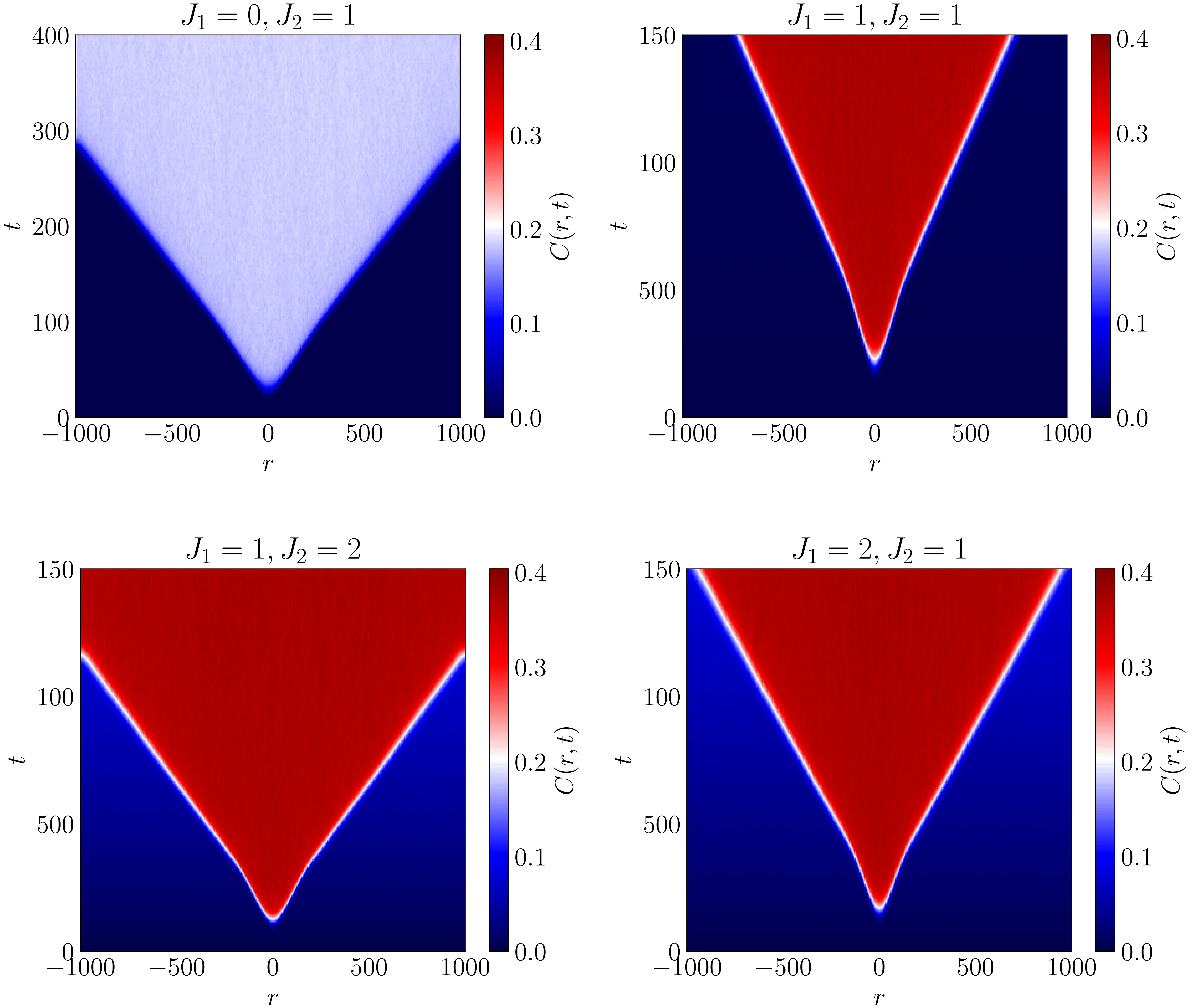
$$\frac{d\mathbf{S}_i}{dt} = J\mathbf{S}_i \ \mathbf{x} \ (\mathbf{S}_{i-1} + \mathbf{S}_{i+1})$$

Averaged over 5000 noisy initial sate realisations.



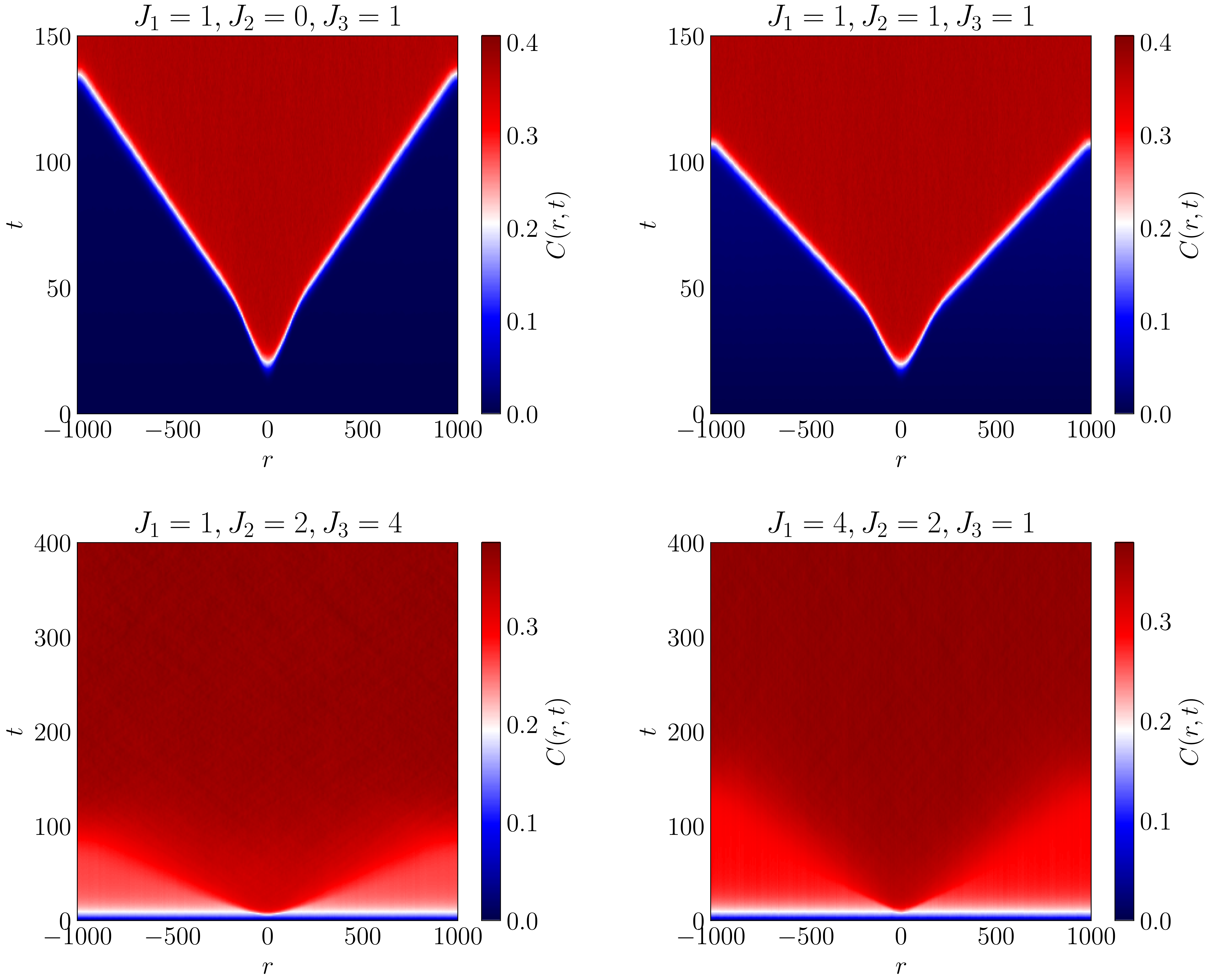
$$\mathscr{H} = -J_1 \sum_{i=0}^{N-1} \mathbf{S_i} \cdot \mathbf{S_{i+1}} - J_2 \sum_{i=0}^{N-1} \mathbf{S_i} \cdot \mathbf{S_{i+2}}$$

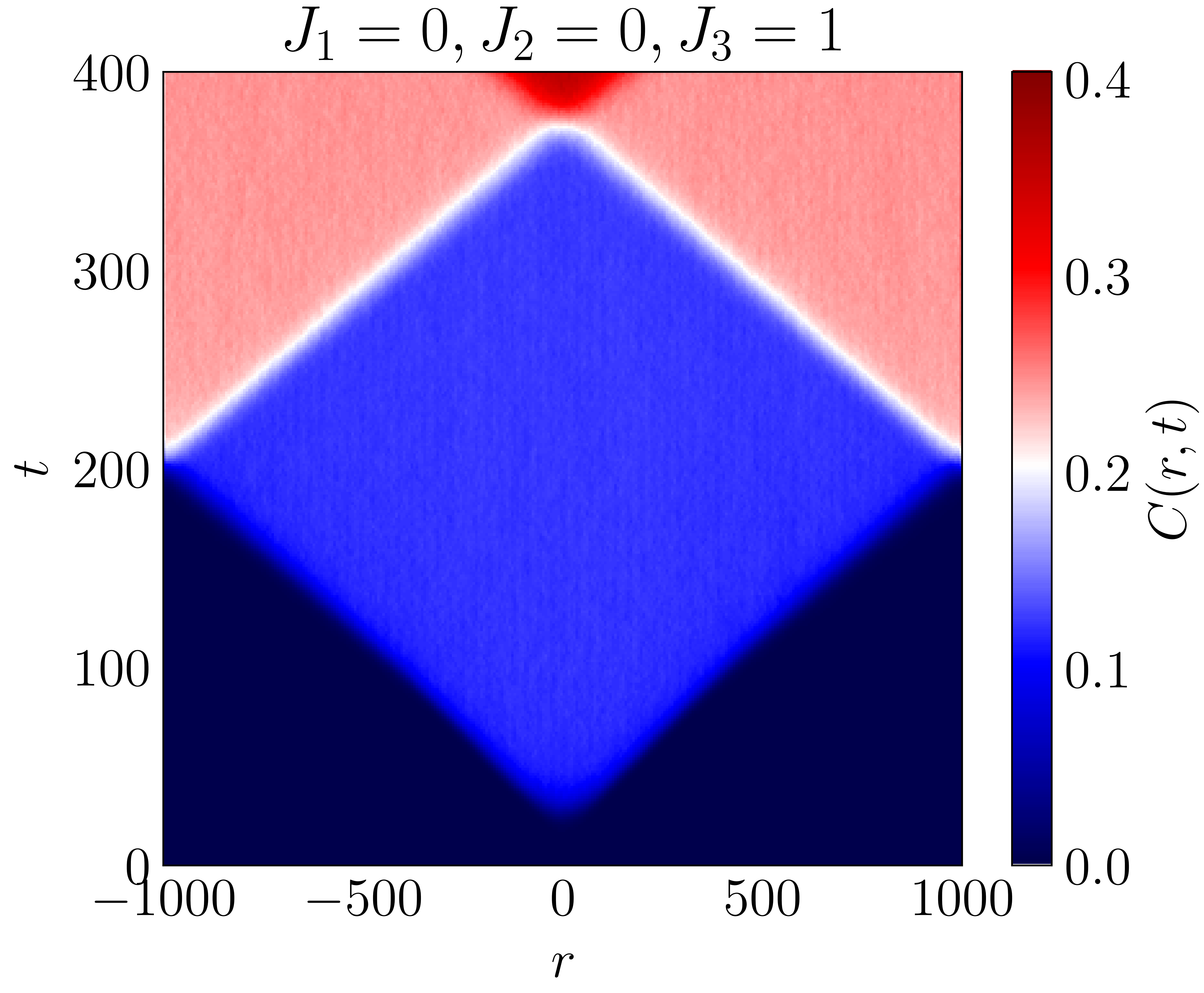
Averaged over 1000 noisy initial sate realisations.



$$\mathscr{H} = -J_1 \sum_{i=0}^{N-1} \mathbf{S_i} \cdot \mathbf{S_{i+1}} - J_2 \sum_{i=0}^{N-1} \mathbf{S_i} \cdot \mathbf{S_{i+2}} - J_3 \sum_{i=0}^{N-1} \mathbf{S_i} \cdot \mathbf{S_{i+3}}$$

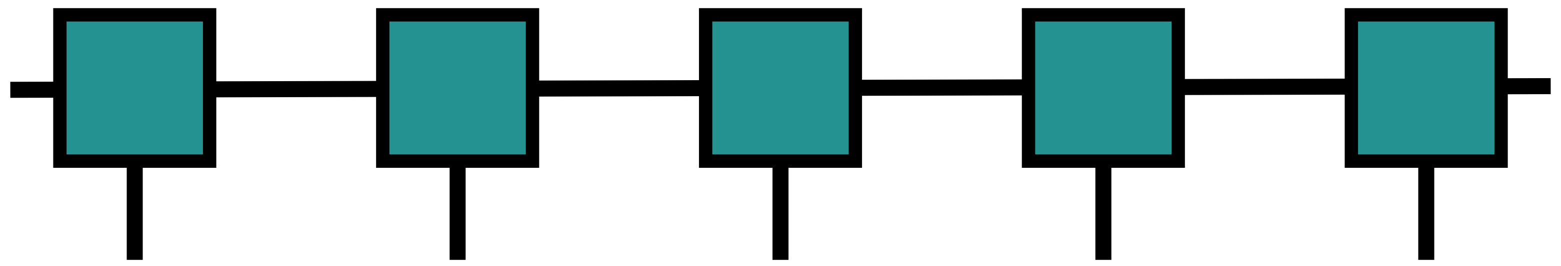
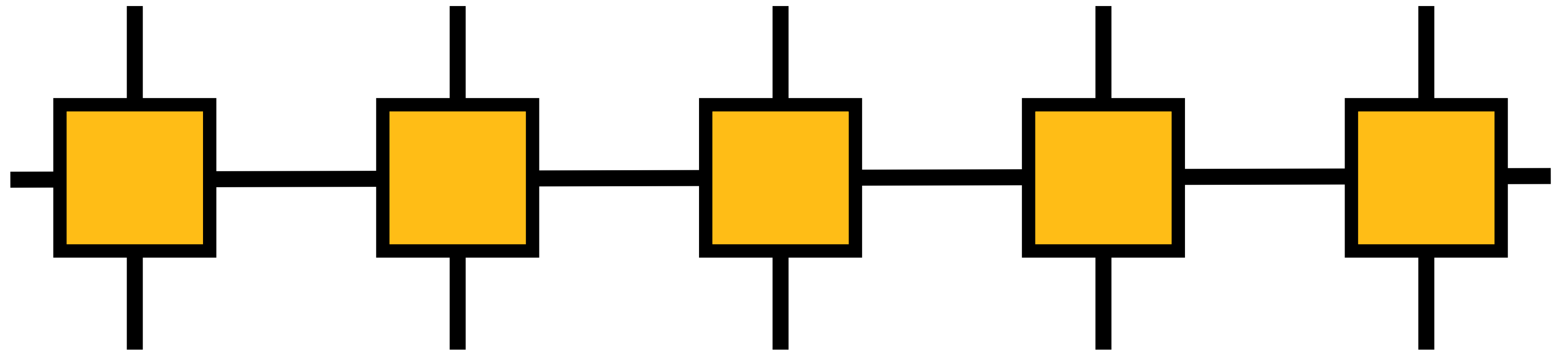
Averaged over 1000 noisy initial sate realisations.

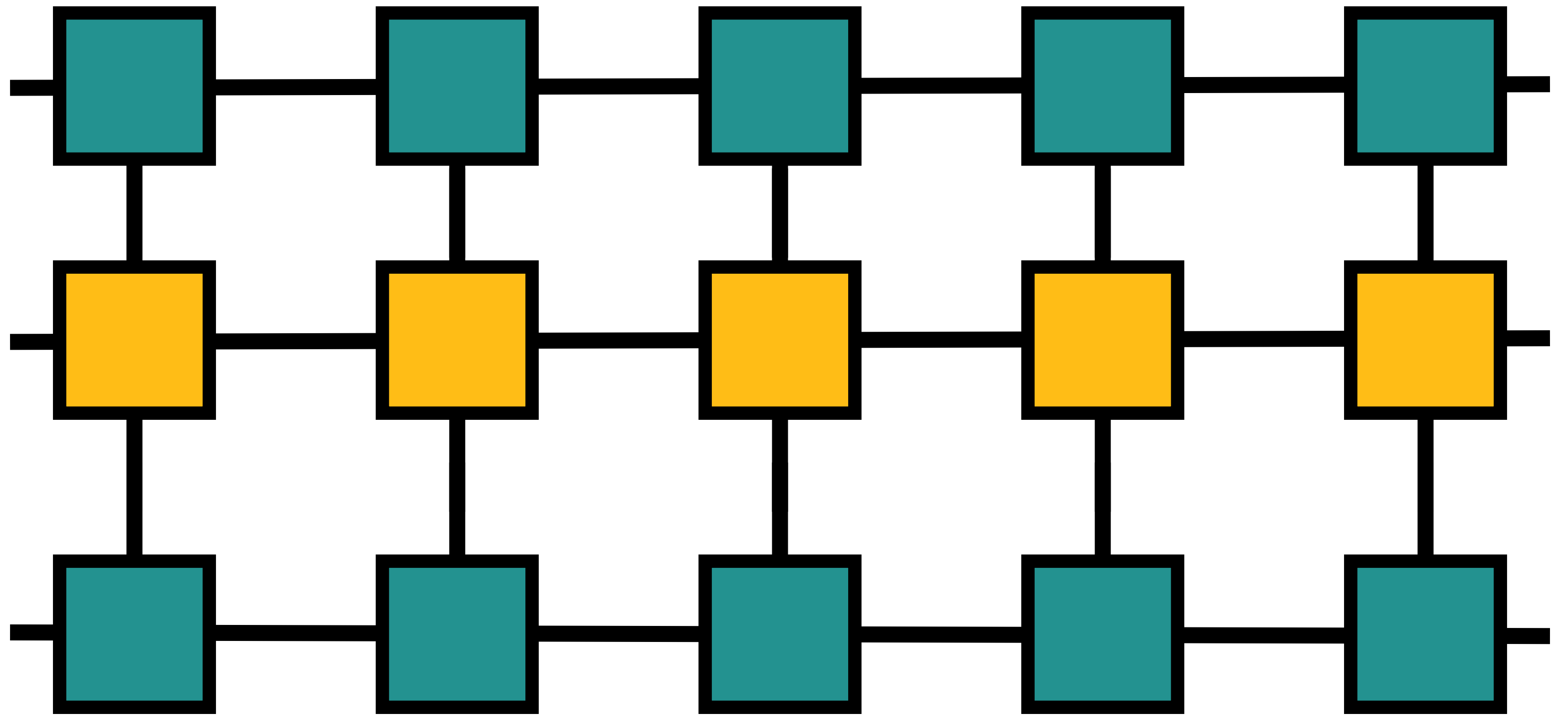



$J_1 = 0, J_2 = 0, J_3 = 1$
400
300
200
100
0
t
−1000
−500
0
500
1000
r
0.4
0.3
0.2
0.1
0.0
$C(r,t)$

Time Evolution of Matrix Product States

$$|\Psi\rangle = \sum_{\sigma} c_{\sigma} |\sigma\rangle \qquad \hat{O} = \sum_{\sigma} c_{\sigma\sigma'} |\sigma\rangle\langle\sigma'|$$

$|\Psi\rangle =$  $\hat{O} =$ 

$\langle\Psi|\,\hat{O}\,|\Psi\rangle =$ 

$U(\delta t)|\Psi\rangle :\qquad e^{-iH\delta t}$ Or $|\Psi(t+\delta t)\rangle$

Time-Evolving Block Decimation

Break the Hamiltonian in parts that commute internally.

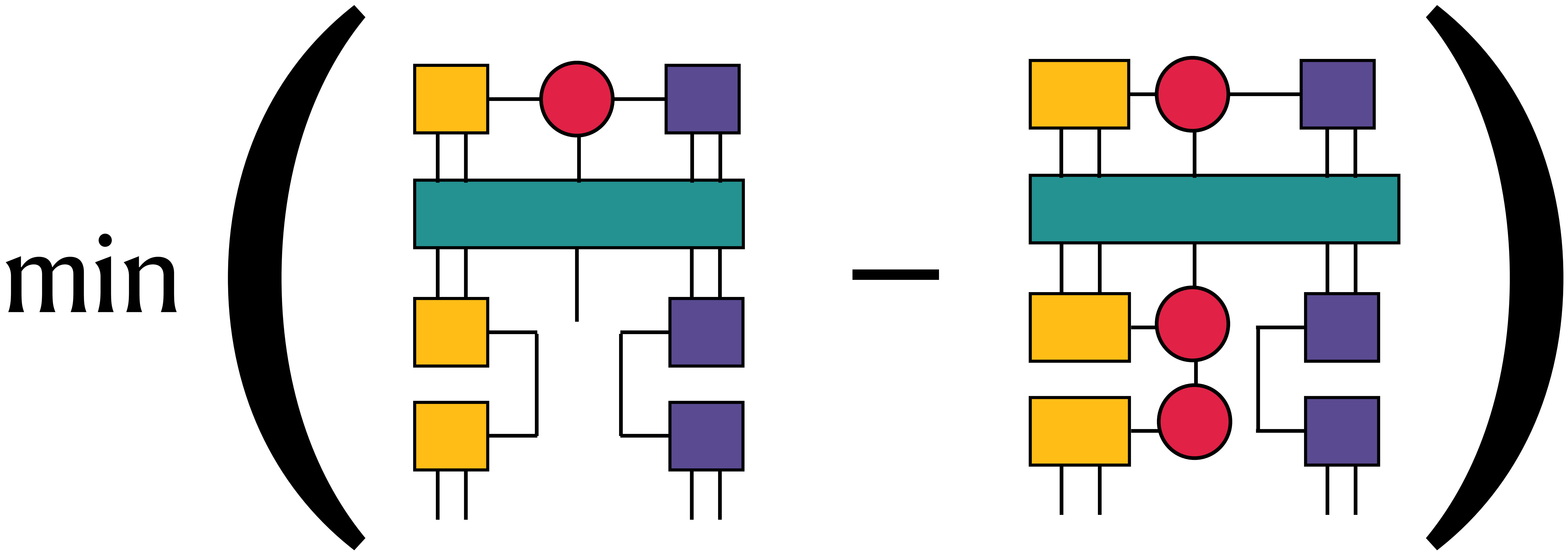
$$H = \sum_N H_N$$

$$U^{TEBD} = \Pi_N e^{-iH_N \delta t}$$

Time-Dependent Variational Principle

Choose a guess MPS from the single-site tangent space of initial MPS and variationally minimise this MPS to obtain the time evolved state.

$$\min \left(\frac{dM^i(t)}{dt} | \partial_i \Psi(M^i) \rangle - \left(\frac{-i}{\hbar} \right) H | \Psi(M^i) \rangle \right) \longrightarrow \frac{d|\Psi(M^i(t))M^i(t)}{dt} = \left(\frac{-i}{\hbar} \right) P_T H | \Psi(M^i) \rangle$$



$$|\psi_1\rangle = W(x,t)V(0,0)|\phi\rangle = e^{\imath Ht}W(x,t)e^{-\imath Ht}V(0,0)|\phi\rangle$$

$$|\psi_2\rangle = V(0,0)W(x,t)|\phi\rangle = V(0,0)e^{\imath Ht}W(x,t)e^{-\imath Ht}|\phi\rangle$$

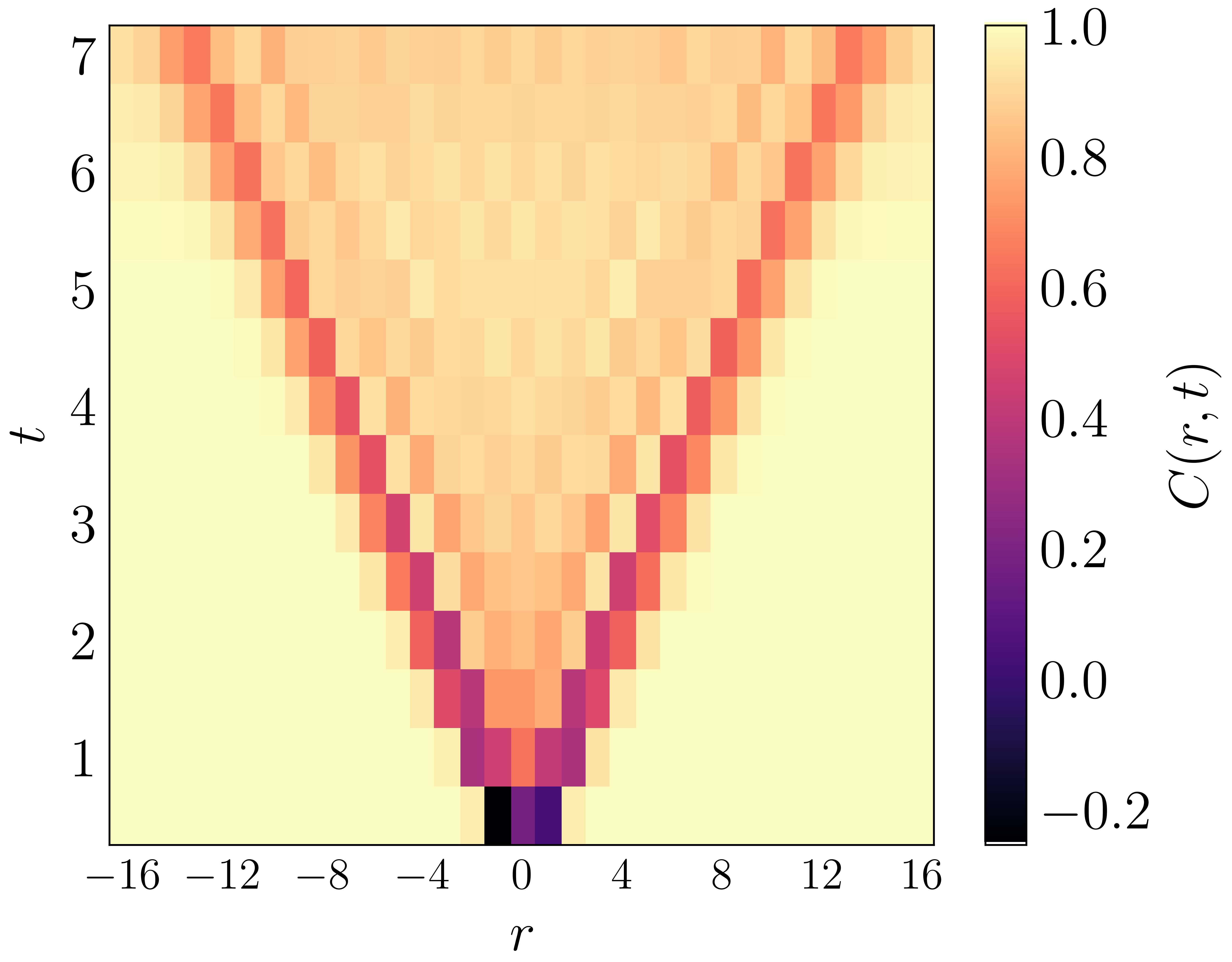
$$\langle W(x,t)V(0,0)W(x,t)V(0,0)\rangle = \langle\psi_2|\psi_1\rangle$$

The Transverse Field Ising Model

$$H = -J\sum_{i=0}^{N-1}\sigma_i^x\sigma_{i+1}^x - g\sum_{i=0}^{N-1}\sigma_i^z$$

$$J = g = 1$$

33 Spins, time steps of 0.5 seconds.



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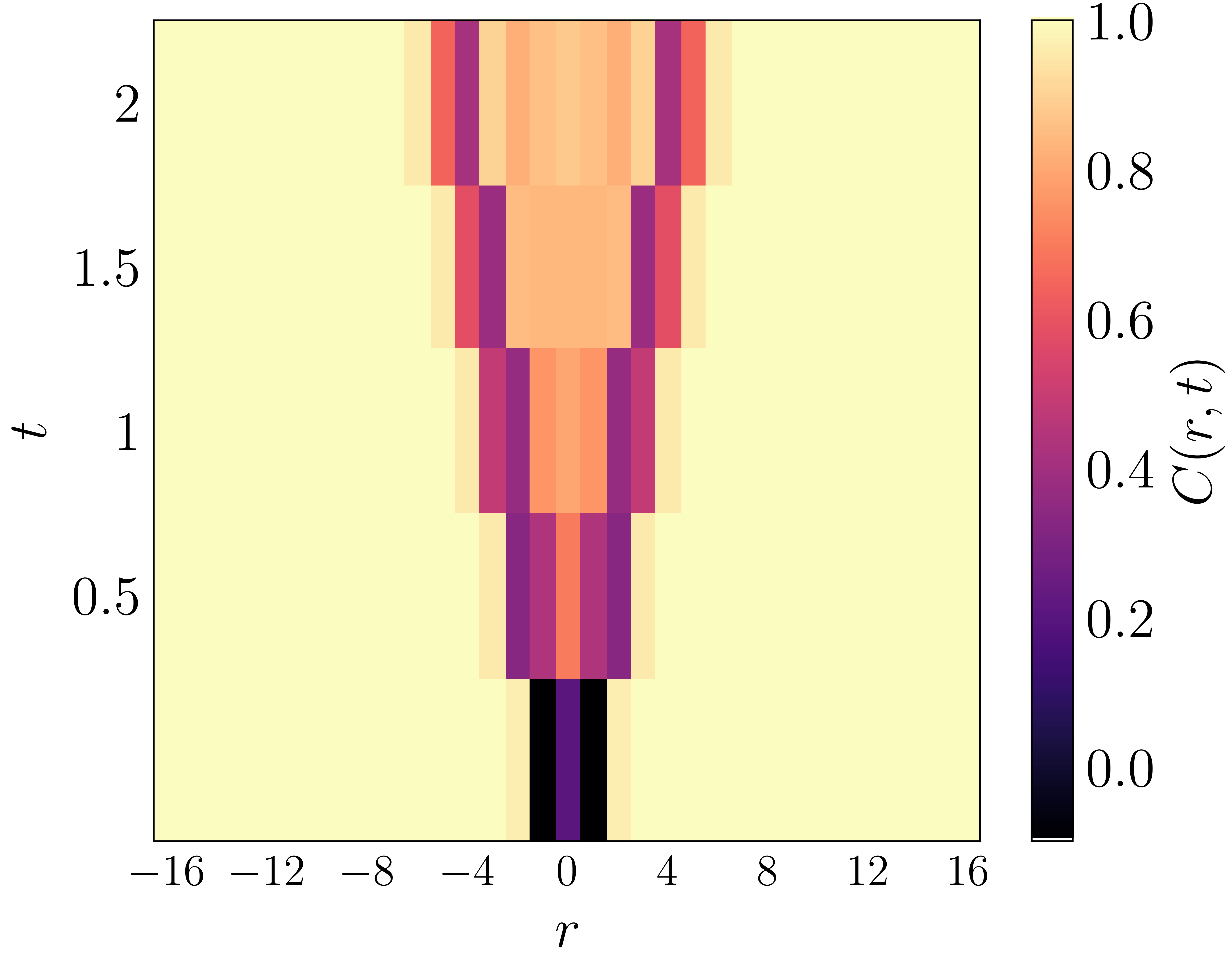
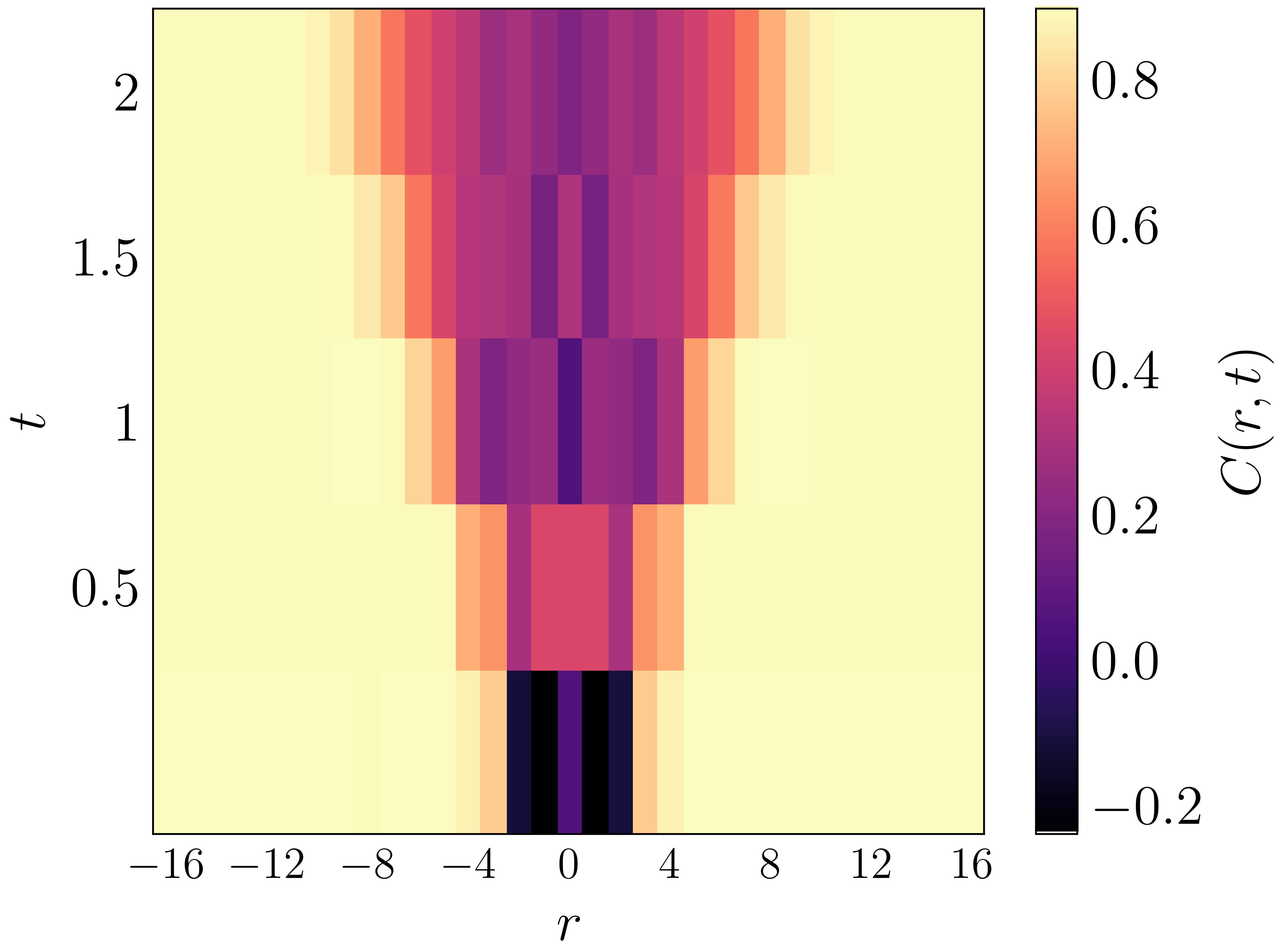


The Transverse Field Ising Model
(J1-J2)

$$H = -\sum_{i=0}^{N-1} \left(J_1 \sigma_i^x \sigma_{i+1}^x + J_2 \sigma_i^x \sigma_{i+2}^x\right) - g \sum_{i=0}^{N-1} \sigma_i^z$$

$$J_1 = J_2 = g = 1$$

33 Spins, time steps of 0.5 seconds.



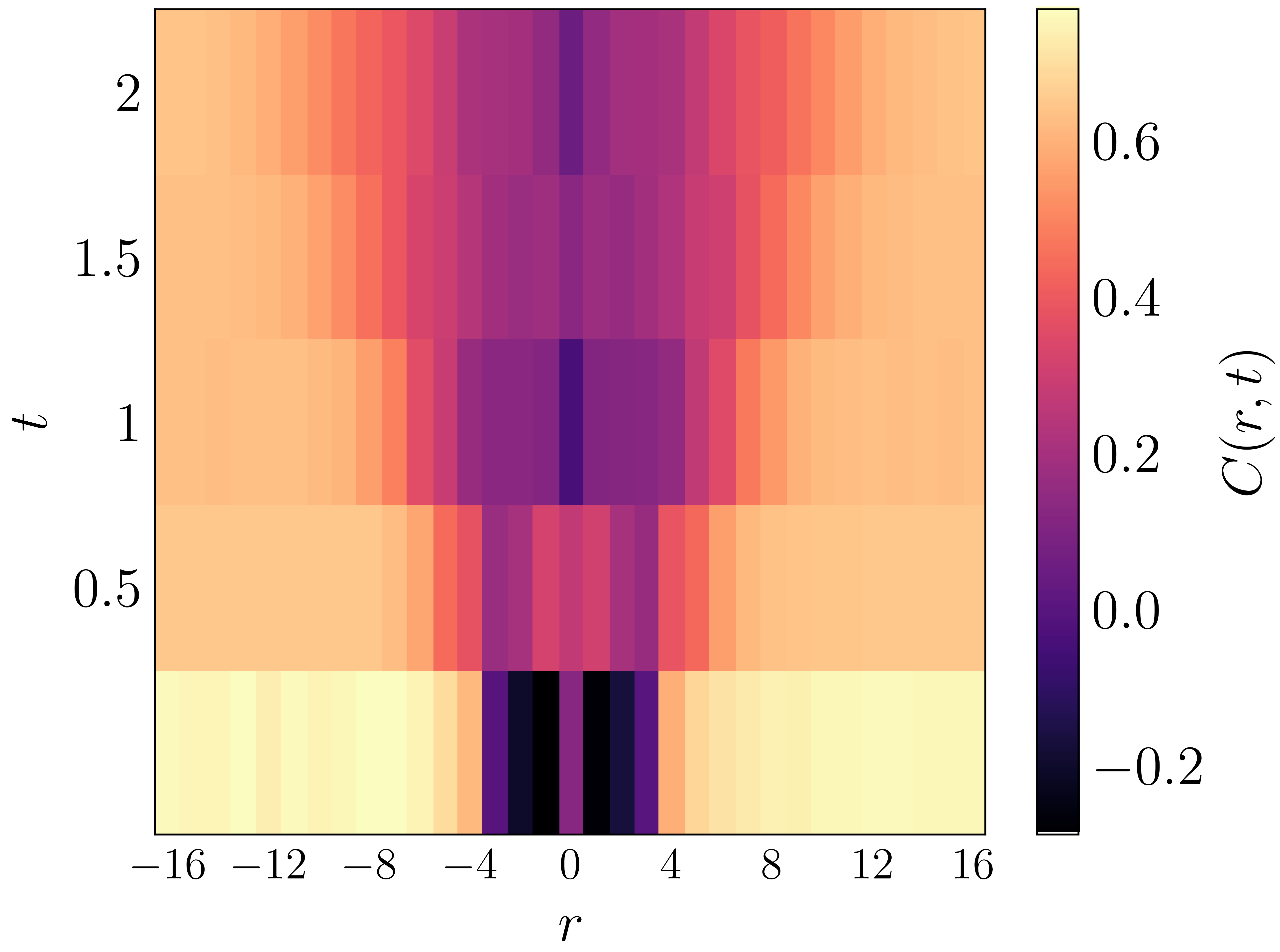
The Transverse Field Ising Model (J1-J2-J3)

$$H = -\sum_{i=0}^{N-1}\left(J_1\sigma_i^x\sigma_{i+1}^x + J_2\sigma_i^x\sigma_{i+2}^x + J_3\sigma_i^x\sigma_{i+3}^x\right)$$

$$-g\sum_{i=0}^{N-1}\sigma_i^z$$

$$J_1 = J_2 = J_3 = g = 1$$

33 Spins, time steps of 0.5 seconds.



Conclusions

Lightcone-like structure of the OTOC (or the decorrelator) emerged for all the systems.

The simulations suffer from averaging over low number of realisations.

For the classical spin system the lightcone structure may be saturated for intermediate-range interacting systems.

A better approach to computing the OTOC for quantum systems might be the MPO evolution methods.